1394 f.

AN INTRODUCTION TO GEOMETRY.

CONTAINING
The moſt uſeful Propoſitions in Euclid,
and other Authors.

Demonſtrated in a clear and eaſy Method, for the Uſe
of Learners.

By **WILLIAM PAYNE.**

⁂

LONDON:
Sold by T. Payne, at the Mews Gate; J. Marks, in
St. Martin's Lane; and M. Hingeston, at Temple
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BRITISH MUSEUM

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TO HIS
ROYAL HIGHNESS
THE
DUKE of *YORK*.

SIR,

THEY who are permitted to prefix the names of Princes to treatiſes of ſcience, generally enjoy the protection of a patron, without fearing the cenſure of a judge.

The honour of approaching your Royal Highneſs, has given me many opportunities of knowing that the work which I now preſume to offer, will not partake of the uſual ſecurity. For as the knowledge which your Royal Highneſs has already acquired of GEOMETRY extends beyond the limits of an introduction, I expect not to inform you; I ſhall be happy if I merit your approbation.

DEDICATION.

An addreſs to ſuch a patron admits no recommendation of the ſcience. It is ſuperfluous to tell your Royal Highneſs that GEOMETRY is the primary and fundamental art of life; that its effects are extended through the principal operations of human ſkill; that it conducts the ſoldier in the field, and the ſeaman in the ocean; that it gives ſtrength to the fortreſs, and elegance to the palace. To your Royal Highneſs all this is already known; GEOMETRY is ſecure of your regard, and your opinion of its uſefulneſs and value has ſufficiently appeared, by the condeſcenſion in which you have been pleaſed to honour one who has ſo little pretenſion to the notice of Princes, as,

SIR,

Your Royal Highneſſes

Moſt obliged,

Moſt obedient,

And moſt humble Servant,

William Payne.

PREFACE.

THE books containing elements of GEOMETRY are ſo numerous, and the names by which they are recommended ſo high in mathematical reputation, that a new treatiſe by an obſcure author, cannot but ſeem to require an apology.

No man ought to increaſe the croud of books, without ſome proſpect of increaſing or facilitating knowledge. I pretend not to boaſt of new diſcoveries in GEOMETRY, my hope is only to make it more readily attainable by the method of demonſtration, which, I believe, to be eaſier than in other books of the ſame kind; tho' I am ſenſible at the ſame time, that every man naturally thinks his own mode of explanation the beſt.

I have allotted a page to each theorem and problem, for the greater eaſe and ſatisfaction of the reader; how well I have ſucceeded, muſt be left to experience.

The Reader is deſired to correct the following Preſs Errors with his Pen.

PAGE 9, line laſt, for 48 read 50; p. 11, line laſt, for EDG read EDF; p. 24, l. 10, for 76 read 77; p. 25 l. 8, for 76 read 77; p. 50, l. 10, for FG read FH; p. 55, l. 8, for BC^2 read BC; p. 66, l. 8, for 62 read 63; p. 72, l. 7, for HI read CI; p. 88, l. 7, for 62 read 63; p. 102, l. 5, for 90, 91, read 91, 93; p. 179, l. 6, for PQ read PR.

INTRODUCTION
TO
GEOMETRY.

1. GEOMETRY contains the nature and properties of lines, angles, ſurfaces, and ſolids.

2. A point is that which has no parts, or magnitude. 1 E Def/1

3. A line is length without breadth. 1 E Def/2

4. A right or ſtrait line, is that which lies evenly between its extremes, or ends. 1 E Def/4

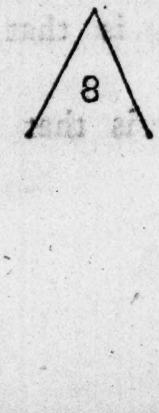
5. A ſurface is that which has length and breadth, but no thickneſs. 1 E Def/5

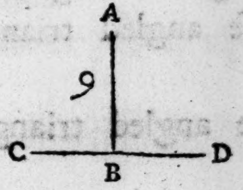
6. A plain ſurface is that which lies evenly, and agrees with a ſtrait line joining any two points in it. 1 E Def/7

7. A ſolid is that which has length, breadth, and thickneſs. 11 E Def/1

8. A plain angle is an opening or corner, made by two ſtrait lines that meet one another. 1 E Def/8

9. When a ſtrait line A B ſtanding upon another C D, makes angles A B C, A B D, on each ſide equal to one another, each of theſe equal angles is called a right angle, and the line A B is ſaid to be perpendicular to the line C D. 1 E Def/10



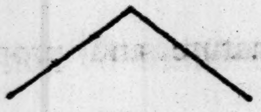


Note. An angle is commonly expreſſed by three letters, that placed at the angular point being always wrote in the middle.

10. An obtuſe angle, is that which is greater than a right angle.

11. An acute angle, is that which is leſs than a right angle.

12. Parallels, are ſuch ſtrait lines in the ſame plain, which if infinitely continued, or produced, will never meet.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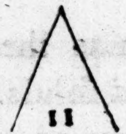
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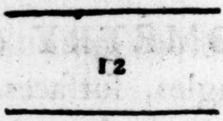
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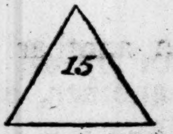
13. A ſpace bounded on all ſides is called a figure.

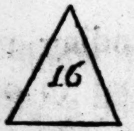
14. A plain triangle, is a figure made by three ſtrait lines.

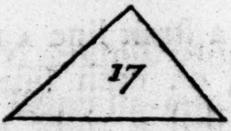
15. An equilateral triangle, is that whoſe three ſides are equal to one another.

16. An iſoſceles triangle, is that which has only two of its ſides equal to one another.

17. A ſcalene triangle, is that which has all its ſides unequal.



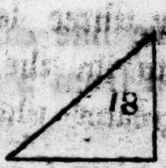


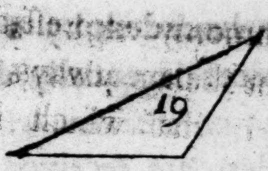


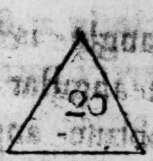
18. A right angled triangle, is that which has a right angle.

19. An obtuſe angled triangle, is that which has an obtuſe angle.

20. An acute angled triangle, is that which has every angle acute.







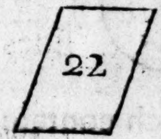
21. A quadrangle, is a figure made by four ſtrait lines.

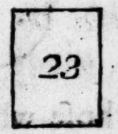
22. A parallelogram, is a quadrangle whoſe oppoſite ſides are parallel.

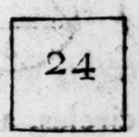
23. A rectangle, is a parallelogram whoſe angles are all right angles.

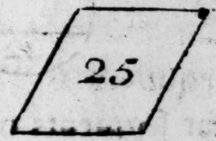
Note. A rectangle is ſaid to be contained by (or within) its adjacent ſides.

24. A ſquare, is a parallelogram whoſe ſides are all equal to one another, and its angles are all right angles.









25. A rhombus is a parallelogram whoſe ſides are all equal to one another, but its angles are not right angles.

All figures having more ſides than four, are in general called polygons.

26. A regular figure, is that whoſe ſides are all equal to one another, and angles all equal to one another.

27. Figures are equiangular, when the angles of the one are equal to the angles of the other, each to each.

28. The baſe of a figure, is the line or plain, upon which it is ſuppoſed to ſtand, or be erected.

29. The altitude of a figure, is a perpendicular drawn from the top or vortex, to the baſe.

30. A circle is a figure, bounded by a curve line called the circumference, every part whereof is equally diſtant from a point within, called the center.

Only ſtrait lines and circles, are received into plain Geometry.

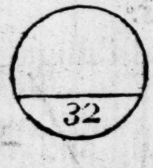
31. The diameter of a circle, is a line paſſing thro' the center and terminated by the circumference. The ſemidiameter is called the radius.

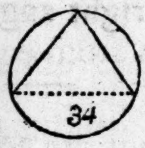
32. A ſegment of a circle, is any part cut off by a ſtrait line.

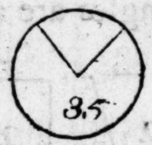
33. Each ſegment made by a diameter is called a ſemicircle.

34. An angle is ſaid to be in a ſegment, when lines are drawn from the baſe of the ſegment to any point in the circumference.

35. A ſector of a circle, is a part contained between two ſemidiameters and the intercepted part of the circumference.







36. Similar ſegments of circles, are thoſe which contain equal angles.

37. Circles are ſaid to touch, when their circumferences meet, but do not cut one another.

38. A tangent to a circle, is a line that touches the circumference, but does not cut it.

39. A line is ſaid to be applied or inſcribed in a circle, when its ends are placed in the circumference.

40. A figure is ſaid to be inſcribed in a circle, when all its angles are placed in the circumference.

41. A circle is ſaid to circumſcribe a figure, when the circumference paſſes thro' all its angular points.

42. A figure is ſaid to be deſcribed about a circle, when every ſide touches the circumference.

43. A circle is ſaid to be inſcribed in a figure, when the circumference touches every ſide of the ſaid figure.

44. One figure is ſaid to be inſcribed in another, when all the angles of the one are placed in the ſides of the other.

45. A theorem, is a truth propoſed to be demonſtrated.

46. A problem, is ſomething propoſed to be done.

47. A corollary, is a truth or theorem which ariſes from a preceeding demonſtration.

Axioms are ſelf-evident truths; as,

48. Things that are equal to the ſame thing, or to equal things, are equal to one another.

49. If equal things be added to equal things, the ſums or wholes will be equal.

50. If equal things be taken from equal things, the remainders will be equal.

51. The halves of equal things are equal, and the doubles of equal things are equal.

52. The whole is greater than a part, and equal to all its parts.

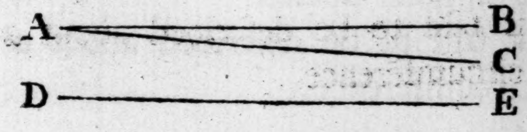
53. Things that mutually agree, or fill equal ſpaces, are equal to one another.

54. All right angles are equal to one another.

55. Equal circles have equal ſemidiameters.

56. Equal things may be taken for one another.

57. Lines that meet one another, are not parallel to one and the ſame line. Thus the lines A B, A C, are not both of them parallel to the line D E.



Poſtulatas, or demands.

58. A line may be drawn from any one point, to any other point.

59. A line may be produced (that is, continued or lengthened) at pleaſure.

60. A circle may be deſcribed about any center, and at any diſtance, or with any radius.

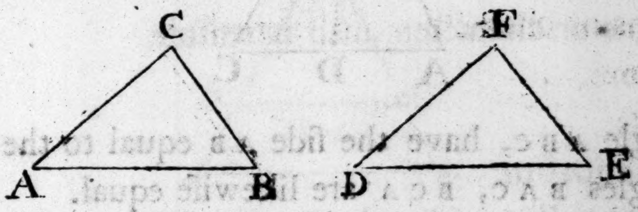
* 61. Of the ſigns or characters uſed hereafter.

+ ſignifies more.
— - - - - leſs.
= - - - - is equal to.
× - - - - into. The rectangle contained by two lines, or multiplication by a number. Thus A B × C D expreſſes a rectangle contained by the lines A B, C D; and A B × n, or n A B is n times the quantity A B, n repreſenting any number whatſoever.

AB^2 ſignifies the ſquare of the line A B.

p. ——— problem.
th. ——— therefore.
Ag. ——— again.
con. ——— conſtruction.
theo. ——— the theorem under conſideration.

62. Triangles that have two ſides and the contained angle of the one, equal to two ſides and the contained angle of the other each to each, are equal, and their remaining parts are equal each to each; namely, the parts oppoſite to equal things. 4. E. I.



Let the triangles ABC, DEF have the ſides BA, AC equal to ED, DF each to each, and the angle BAC equal to the angle EDF: then the triangles are equal, the ſide BC is equal to the ſide EF, and the angles ABC, ACB are equal to DEF, DFE each to each.

For conceive the triangle ABC applied to DEF, with the point A on the point D, and the line AB on the line DE; the point B will then agree with E, becauſe AB is equal to DE, the line AC will agree with the line DF, becauſe the angle BAC is equal to EDF, and the point C muſt agree with F, ſince AC is equal to DF. Again, as the points B, C agree with the points E, F, the line BC muſt agree with the line EF —— —— —— —— 4

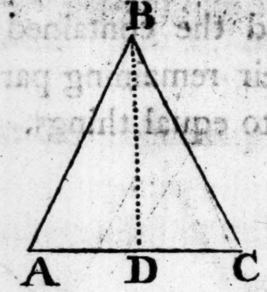
Th. The triangle ABC is equal to DEF —— —— 53

—— The ſide BC is equal to the ſide EF —— —— 53

And the angles ABC, ACB are equal to DEF, DFE each to each 53

63. In

.5 b.1. 63. In triangles, oppoſite to equal ſides are equal angles.



Let the triangle A B C, have the ſide A B equal to the ſide B C: then the oppoſite angles B A C, B C A are likewiſe equal.

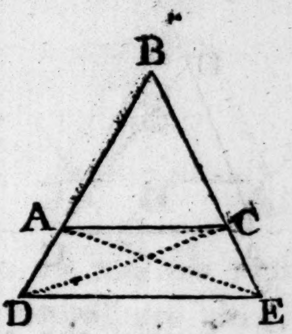
For conceive the line B D drawn to biſect the angle A B C, or to divide it equally.

Now the ſide A B is equal to the ſide B C —— theo.
The line B D is common to both —— —— ——
Th. the ſides A B, B D are equal to C B, B D each to each. above
And the contained angles A B D, D B C are equal —— above
Th. the angle B A C is equal to B C A —— —— 62

63. Other-

63. Otherwiſe

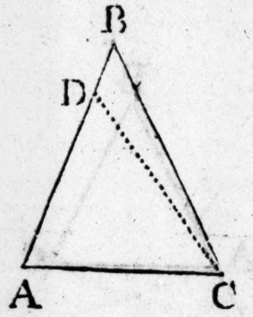
In triangles, oppoſite to equal ſides A B, B C are equal angles B A C, B C A.



For produce the equal ſides A B, B C to D and E, aſſume the point D, make B E equal to B D, or C E equal to A D by problem 2, and join A E, C D.

Now the ſides A B, B E are equal to C B, B D each to each. con.
And the contained angle A B E is common —— ——
Th. the ſide E A is equal to the ſide D C —— —— 62
— the angle C E A is equal to A D C —— —— 62
And the angle B A E is equal to B C D —— 62
Ag. the ſides C E, E A are equal to A D, D C each to each. above
And the contained angles C E A, A D C are equal —— above
Th. the angle C A E is equal to A C D —— 62
Th. the angle B A C is equal to B C A —— 50. ~~48~~

6.E.I. 64. In triangles, oppoſite to equal angles are equal ſides.

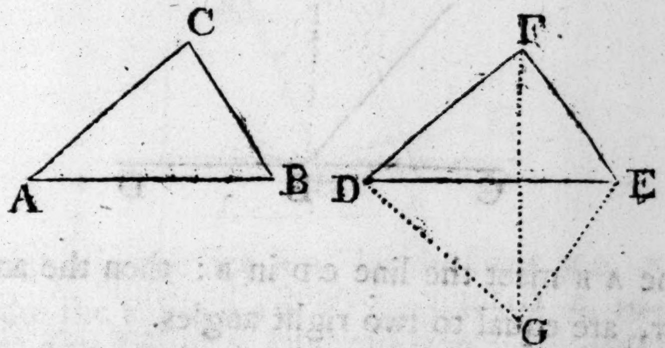


Let A B C be a triangle, having the angle A C B equal to the angle B A C: then their oppoſite ſides A B, B C are alſo equal.

For if not, let A B be the greater, and in the ſame take A D equal to C B by problem 2, and join C D.

Now the ſides C A, A D are equal to A C, C B each to each. con.
And the contained angles C A D, A C B are equal —— theo.
Th. the triangles C A D, A C B are equal —— —— 62
Which is impoſſible —— —— —— 52
Th. A B is not greater than B C —— —— ——
By the ſame reaſon A B is not leſs than B C —— ——
Th. the ſides A B, B C muſt be equal —— ——

65. Triangles whoſe ſides are equal to one another each to each, are equiangular, having thoſe angles equal which are oppoſite to equal ſides. 8 El.



Let the triangles A B C, D E F have the ſides A B, B C, C A equal to D E, E F, F D each to each: then oppoſite to the equal ſides are equal angles.

For make the angle E D G equal to the angle B A C by problem 9, the ſide D G equal to the ſide A C or D F by problem 2, and join E G, G F.

Now the ſides E D, D G are equal to B A, A C each to each. con.

And the contained angles E D G, B A C are equal —— con.

Th. the angles D G E, A C B are equal, the angles G E D, C B A are equal, and the ſides E G, B C are equal —— 62

Th. the ſide E G is equal to the ſide E F —— —— 48

Ag. the angles D G F, D F G are equal, and F G E, G F E are equal —— —— —— —— 63

Th. the angles D G E, D F E are equal. —— —— 49

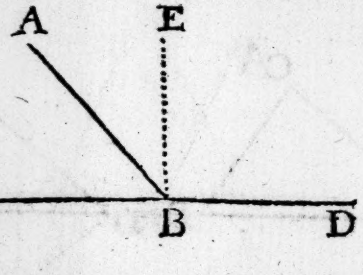
And the ſides D G, G E are equal to D F, F E each to each. above

Th. the angles E D G, E D F are equal, and G E D, F E D are equal —— —— —— —— 62

Th. the angle A C B = D F E, B A C = E D ~~G~~ F, and C B A = F E D. 48

66. One

13. 6.5. 66. One line meeting another otherwiſe than at the end, makes with it two angles, which taken together, are equal to two right angles.



Let the line AB meet the line CD in B: then the angles CBA, ABD both together, are equal to two right angles.

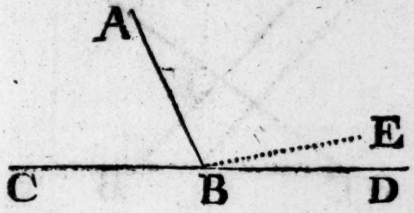
For if the angles CBA, ABD are equal, each is a right angle by the 9th; and if not, draw BE at right angles to CD by problem 6.

Now the angle CBA + ABE ——— = CBE ——— 52
Th. ——— CBA + ABE + EBD = CBE + EBD ——— 49
Again, the angle ——— ABE + EBD = ABD ——— 52
Th. ——— CBA + ABD ——— = CBE + EBD ——— 56
But CBE, EBD are right angles ——— ——— con.

Th. the angle CBA + ABD ——— = two right angles 56

67. Corollary. All the angles which can be made by lines proceeding from a given point taken together, are equal to four right angles.

68. If from a point in any line, two lines be drawn contrary ways making the adjacent angles equal to two right angles, theſe lines will have the ſame direction and fall into one ſtrait line. 14. E. 1.

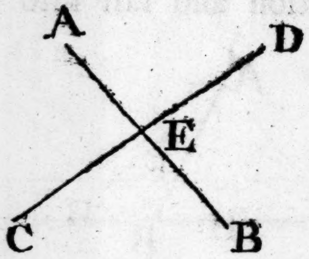


From the point B in the line A B, let the lines B C, B D drawn contrary ways, make the angle $CBA + ABD$ equal to two right angles: then C B D is a ſtrait line.

For if not, produce C B to E.

Now the angle $CBA + ABE =$ two right angles —— 66
And —— $CBA + ABD =$ two right angles —— theo.
Th. —— $CBA + ABE = CBA + ABD$ —— 48
Th. —— —— $ABE = ABD$ —— 50
Which is impoſſible —— —— —— 52
Th. C B D muſt be a ſtrait line. —— ——

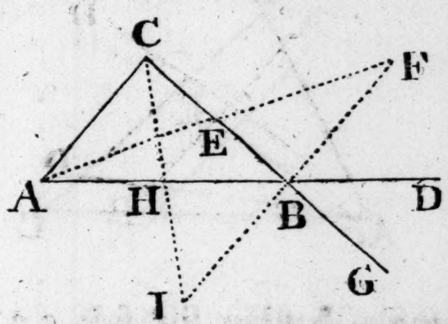
15. E. 1. **69. The oppoſite angles made by two lines cutting each other, are equal to one another.**



Let the lines AB, CD cut each other in E; then the angle AEC is equal to DEB, and the angle AED is equal to BEC.

For the angle AEC + AED = two right angles ——— 66
And the angle AED + DEB = two right angles ——— 66
Th. the angle AEC + AED = AED + DEB ——— 48
Th. the angle AEC ——— = ——— DEB ——— 50
Ag. the angle AED + DEB = two right angles ——— 66
And the angle DEB + BEC = two right angles ——— 66
Th. the angle AED + DEB = DEB + BEC ——— 48
Th. the angle AED ——— = ——— BEC ——— 50

70. **If any ſide of a triangle is produced, the outward angle will be** 16.E.1.
greater than either of the inward and remote angles.



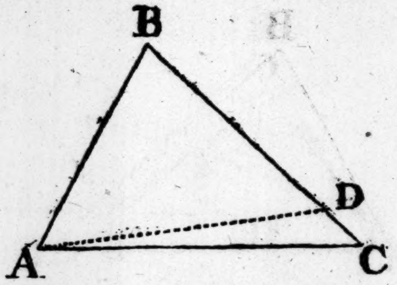
Let A B C be a triangle, whoſe ſides A B is continued to D: then the angle C B D is greater than either of the angles A C B, B A C.

For biſect B C in E by problem 5, draw A E and produce it to F, make E F equal to E A, join F B, and continue C B to G.

Now the ſides C E, E A are equal to B E, E F each to each. con.
And the contained angles C E A, B E F are equal —— 69
Th. the angle A C E is equal to E B F —— —— 62
But the angle C B D is greater than E B F —— 52
Th. the angle C B D is greater than A C E, or A C B —— 56
By the ſame reaſon the angle A B G is greater than B A C ——
And the angles C B D, A B G are equal —— —— 69
Th. the angle C B D is greater than B A C —— —— 56

71. In

18 E. 1. 71. **In triangles, oppoſite to the greater ſide is the greater angle.**

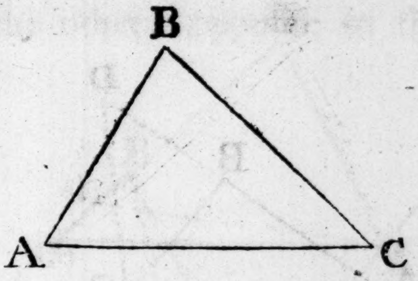


Let A B C be a triangle, having the ſide B C greater than the ſide B A, then the angle B A C is greater than the angle B C A.

For in B C take B D equal to B A by problem 3, and join A D.

Now the ſide B D is equal to the ſide B A —— con.
Th. the angle B A D is equal to B D A —— —— 63
But the angle B D A is greater than B C A —— —— 70
Th. the angle B A D is greater than B C A —— 56
Th. the angle B A C is much greater than B C A —— —

72. In triangles, oppoſite to the greater angle is the greater ſide. 19. l. 1.



Let the triangle A B C, have the angle A greater than the angle C, then the ſide B C is greater than the ſide B A.

For if the ſide B C was leſs than the ſide B A, the angle A would be leſs than the angle C ——— ——— ——— 71

But it is not leſs ——— ——— ——— theo.

Th. B C is not leſs than B A ——— ——— ———

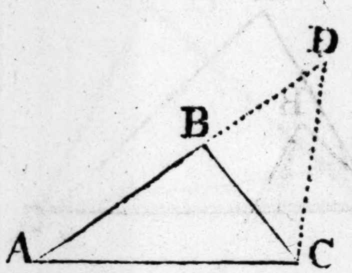
Again, if the ſide B C was equal to the ſide B A, the angle A would be equal to the angle C ——— ——— 63

But it is not equal ——— ——— ——— theo.

Th. B C is not equal to B A ——— ——— ———

Conſequently, B C muſt be greater than B A ——— ———

20. b. 1. 73. **Any two ſides of a triangle taken together, are greater than the remaining ſide.**

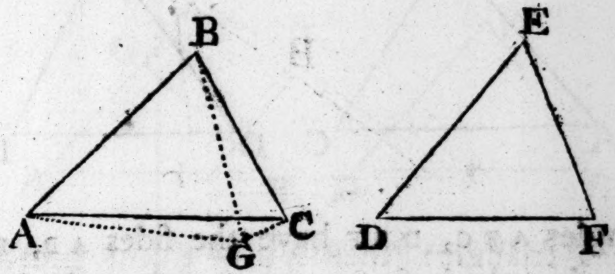


Let A B C be a triangle: then A B + B C is greater than C A, B C + C A is greater than A B, and C A + A B is greater than B C.

For produce A B to D, make B D equal to B C, and join C D.

Now the ſide B D is equal to the ſide B C —— —— con.
Th. the angle B C D is equal to B D C —— —— 63
But the angle A C D is greater than B C D —— —— 52
Th. the angle A C D is greater than B D C —— —— 56
Th. the ſide A D is greater than the ſide C A —— 72
But A B + B C is equal to A D —— —— 49
Th. A B + B C is greater than C A —— —— 56
Alſo B C + C A is greater than A B } by the ſame reaſon.
And C A + A B is greater than B C }

74. If triangles have two ſides of the one equal to two ſides of the other each to each, but the contained angle in the one greater than the contained angle in the other: oppoſite to the greater angle is the greater baſe. 24. E. 1.



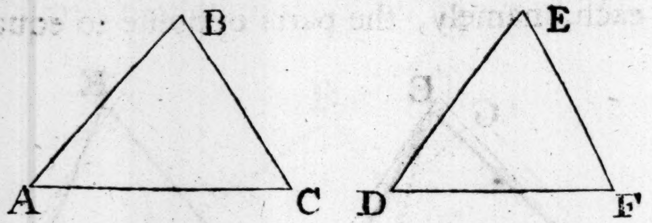
Let the triangles A B C, D E F have the ſides A B, B C equal to D E, E F each to each, but the angle A B C greater than the angle D E F; then the baſe A C is greater than the baſe D F.

For make the angle A B G equal to the angle D E F by problem 9, and the ſide B G equal to the ſide E F by problem 3, and join A G, G C.

Now the ſide B G is equal to B C ——— ——— 48
Th. the angle B C G is equal to B G C ——— ——— 63
But the angle B C G is greater than A C G ——— 52
Th. the angle B G C is greater than A C G ——— 56
Th. the angle A G C is much greater than A C G ——— 52
Th. the ſide A C is greater than A G ——— ——— 72
But A G and D F are equal ——— ——— ——— 62
Th. the baſe A C is greater than the baſe D F ——— 56

75. If

25. b. 1. 75. **If triangles have two ſides of the one equal to two ſides of the other each to each, but the baſe of the one greater than the baſe of the other: oppoſite to the greater baſe is the greater angle.**



Let the triangles A B C, D E F have the ſides A B, B C equal to D E, E F each to each, but the baſe A C greater than the baſe D F: then the angle B is greater than the angle E.

For if the angle B was equal to the angle E, the baſe A C would be equal to the baſe D F —— —— —— 62.

But they are not equal —— —— —— theo.

Th. the angle B is not equal to the angle E —— ——

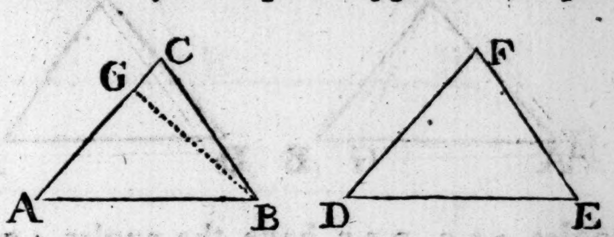
Again, if the angle B was leſs than the angle E, the baſe A C would be leſs than the baſe D F —— —— 74

But it is not leſs —— —— —— theo.

Th. the angle B cannot be leſs than E —— ——

Conſequently, the angle B muſt be greater than the angle E ——

76. If triangles have two angles of the one equal to two angles of the other each to each, and one ſide of the one equal to its correſponding ſide in the other; the remaining parts of thoſe triangles will alſo be equal each to each, namely, the parts oppoſite to equal things. 26.E1.



Let the triangles ABC, DEF have the angles ABC, CAB equal to DEF, FDE each to each, and the ſides AB, DE equal, that are between the equal angles: then the ſides AC, CB are equal to DF, FE each to each, and the angle ACB is equal to DFE.

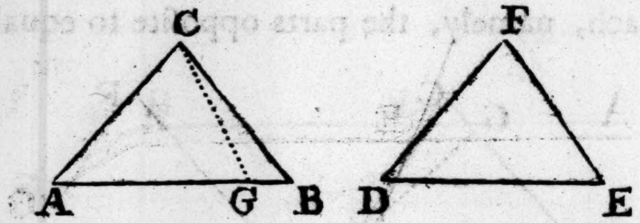
For if AC and DF are not equal, let AC be the greater, and in it take AG equal to DF by problem 3, and join GB.

Now the ſides BA, AG are equal to ED, DF each to each. con.
And the angle BAG is equal to EDF —— theo.
Th. the angle ABG is equal to DEF —— 62
But the angle ABC is equal to DEF —— theo.
Th. the angles ABG, ABC are equal —— 48
Which is impoſſible —— —— 52
Th. the ſides AC, DF muſt be equal —— ——
Th. the ſides CB, FE are equal —— —— 62
And the angle ACB is equal to DFE —— 62

76. Caſe

76. Caſe 2.

When the equal ſides are oppoſite to equal angles.

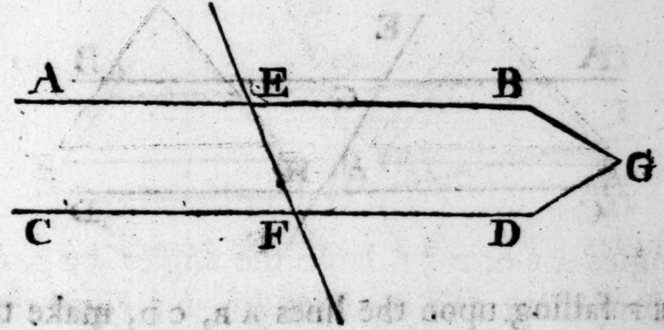


Let the triangles A B C, D E F have the angles A B C, C A B equal to D E F, F D E each to each, and the ſides A C, D F equal that are oppoſite to equal angles: then the ſides A B, B C are equal to D E, E F each to each, and the angle A C B is equal to D F E.

For if A B and D E are unequal, let A B be the greater, in which make A G equal to D E, and join C G.

Now the ſides C A, A G are equal to F D, D E each to each. con.
And the contained angles C A G, F D E are equal ——— theo.
Th. the angle A G C is equal to D E F ——— 62
But the angle A B C is equal to D E F ——— theo.
Th. the angles A G C, A B C are equal ——— 48
Which is impoſſible ——— ——— 70
Th. the ſides A B, D E muſt be equal ——— ———
Th. the ſides B C, E F are equal ——— ——— 62
And the angle A C B is equal to D F E ——— 62

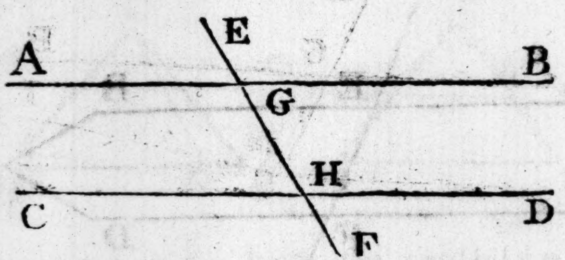
77. If a ſtrait line falling upon two ſtrait lines, makes the alternate angles equal to one another, thoſe two lines are parallels. 27 l.s.



Let the line E F falling upon the lines A B, C D, make the alternate angles A E F, E F D equal to each other: then the lines A B, C D are parallels.

For if not, let them be continued till they meet at G. —— 59
Now the angle A E F is equal to E F G —— theo.
And the angle A E F is greater than E F G —— 70
Which is abſurd, for things cannot be equal and unequal. ——
Th. the lines A B, C D continued will not meet. —— ——
Th. A B, C D are parallels. —— —— 12

28 l. 78. If a ſtrait line falling upon two ſtrait lines, makes the outward angles equal to the inward and oppoſite angles on the ſame ſide, thoſe two lines are parallels.



Let the line EF falling upon the lines AB, CD, make the angle EGB equal to the angle GHD; then AB, CD are parallels.

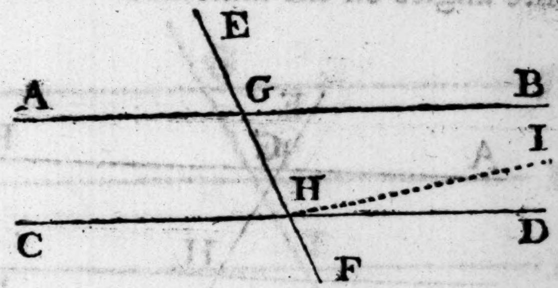
For the angle EGB is equal to AGH ——— 69
And the angle EGB is equal to GHD. ——— theo.
Th. the angle AGH is equal to GHD ——— ——— 48
And they are alternate angles ——— ——— ———
Th. AB, CD are parallels ——— ——— 77. ~~76~~

79. Corollary.

If a ſtrait line falling upon two ſtrait lines, makes the inward angles on the ſame ſide equal to two right angles, thoſe two lines are parallels.

For if the angle BGH + GHD = two right angles ——— theo.
Alſo the angle AGH + BGH = two right angles ——— 66
Th. the angle AGH + BGH = BGH + GHD ——— 48
Th. the angle AGH ——— = ——— GHD ——— 50
Th. AB, CD are parallels. ——— ——— 77

80. A ſtrait line falling upon parallels, makes the alternate angles equal to one another. 29 l. 1.

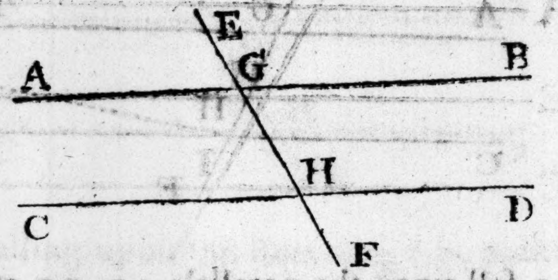


Let the line E F fall upon the parallels A B, C D at G and H; then the alternate angles A G H, G H D are equal.

For if not, make the angle G H I equal to A G H ——— p. 9
Now the angles A G H, G H I are equal ——— ——— con.
And they are alternate angles ——— ——— ———
Th. the lines A B, H I are parallels ——— ——— ~~76~~ 77
But the lines A B, C D are parallels ——— ——— theo.
Th. the lines H I, C D meeting in H, are both parallel to the line A B ——— ——— ——— ———
Which is impoſſible ——— ——— ——— 57
Th. the angles A G H, G H D muſt be equal ——— ———

81. Corollary.

A line falling upon parallels, makes the outward angles equal to the inward and oppoſite angles on the ſame ſide.



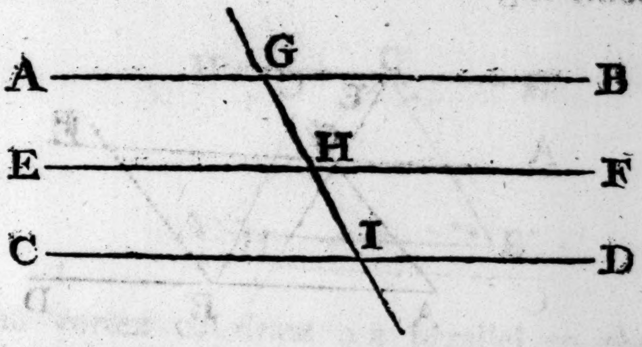
For the angle E G B is equal to A G H ——— 69
And the angle G H D is equal to A G H ——— 80
Th. the angle E G B is equal to G H D ——— ——— 48

82. Corollary.

A line falling upon parallels, makes the two internal angles on the ſame ſide equal to two right angles.

For the angle A G H + H G B = two right angles ——— 66
Th. the angle G H D + H G B = two right angles ——— 56

83. Lines that are parallel to the ſame line, are parallel to one another. 30. l. 1.



Let each of the lines A B, C D be parallel to the line E F; then A B, C D are parallels.

For let the ſtrait line G H I fall upon them.

Now the angle A G I is equal to G H F —— —— 80

And the angle G I D is equal to G H F —— —— 81

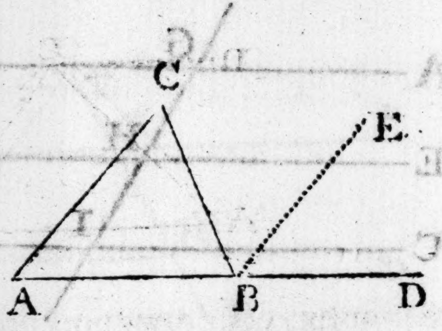
Th. the angle A G I is equal to G I D —— —— 48

And they are alternate angles —— —— ——

Th. A B, C D are parallels —— —— —— 77

E 2 84. If

32. E. §. 84. If any ſide of a triangle is produced, the outward angle will be equal to both the inward and remote angles: and the three angles of the triangle altogether, are equal to two right angles.



Let the triangle A B C have the ſide A B produced to D: then the angle C B D is equal to both the angles A C B, B A C, and the three angles of the triangle are equal to two right angles.

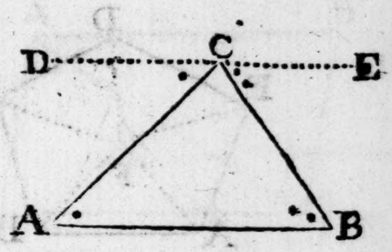
For draw B E parallel to A C ——— p. 10
Now the angle C B E ——— = A C B ——— 80
And the angle E B D ——— = B A C ——— 81
Th. the angle C B D ——— = A C B + B A C ——— 49

Secondly,

The angle C B D + C B A ——— = A C B + B A C + C B A. 49
But the angle C B D + C B A ——— = two right angles. 66
Th. the angle A C B + B A C + C B A = two right angles. 48

Again,

The three angles of a plain triangle ABC altogether, are equal to two right angles.



For thro' the vortex C, draw DE parallel to the baſe AB, by problem 10.

Now the angle ABC ———— ———— = BCE ———— — 80

Th. the angle ABC + BCA ———— = ECA ———— — 49

Ag. the angle ———— ———— CAB = ACD ———— — 80

Th. the angle ABC + BCA + CAB = ECA + ACD — 49

But two right angles ———— ———— = ECA + ACD — 66

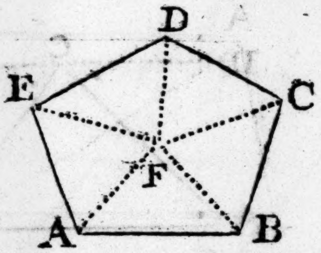
Th. the angle ABC + BCA + CAB = two right angles. 48

85. Cor. If two angles of one triangle be equal to two angles of another, the remaining angles are alſo equal.

86. Cor. In right angled triangles, the right angle is equal to the ſum of both the acute angles.

87. All

Cor. 1. 32 El. 1. 87. All the angles of a figure (when they are all inward) are equal to twice as many right angles, excepting four, as the figure has ſides.



Let A B C D E be a right lined figure; then the ſum of all its angles A, B, C, D, E are equal to twice as many right angles, excepting four, as the figure has ſides.

For aſſume the point F within the figure, and join F A, F B, F C, &c.

Now the figure is divided into as many triangles as it has ſides. Con.

And each triangle contains two right angles. ——— 84

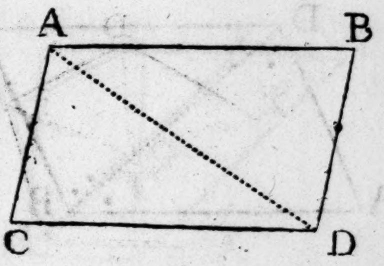
Therefore, all the triangles contain twice as many right angles as the figure has ſides ——— ——— ———

But all the angles at the point F taken together, are equal to four right angles ——— ——— ——— 67

And the remaining angles of the triangles compoſe the angles of the figure ——— ——— ——— ———

Conſequently, all the angles of the figure taken together, are equal to twice as many right angles, excepting four, as the figure has ſides. ——— ——— ——— ———

88. If the ends of two equal and parallel lines, be joined by lines not cutting one another: theſe lines will alſo be equal and parallel. 33. l.1.



Let the equal and parallel lines A B, C D be joined by A C, B D: then the lines A C, B D are equal and parallel.

For draw the diagonal A D.

Now the line A B is equal to the line C D. — theo.

Th. the ſides B A, A D are equal to C D, D A each to each —

And the contained angles B A D, C D A are equal — 80

Th. the line A C is equal to the line B D — 62

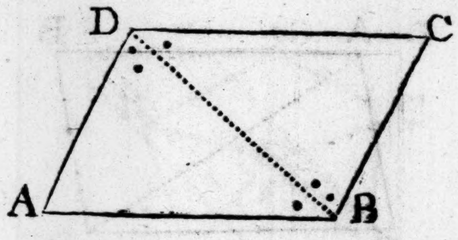
Alſo the angle A D B is equal to D A C — — 62

And they are alternate angles — — —

Th. A C, B D are parallels — — 77

89. The

34 E. 1. 89. **The oppoſite ſides, and oppoſite angles of a parallelogram are equal, and a diagonal line divides it into equal parts.**



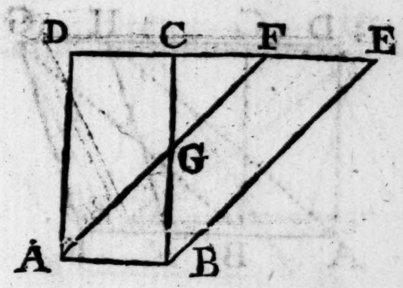
Let A B C D be a parallelogram, and draw the diagonal B D: I ſay, its oppoſite ſides, and oppoſite angles are equal, and the triangle B A D is equal to D C B.

For the angles A B D, A D B are equal to C D B, C B D each to each. 80
And the correſponding ſide B D is common ——— ——
Th. the ſides B A, A D are equal to D C, C B each to each. 76
And the angle B A D is equal to D C B ——— ——— 76
Th. the triangle B A D is equal to D C B ——— 62

90. Corollary.

A parallelogram that has a right angle, is a rectangle ——— 23

91. Parallelograms ſtanding upon the ſame baſe, and between the ſame parallels, are equal to one another. 35. l. 1.

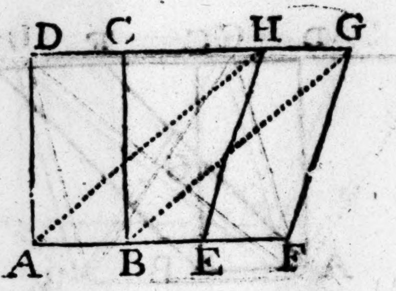


Let A B C D, A B E F be parallelograms, ſtanding upon the baſe A B, and between the parallels A B, D E : then A B C D is equal to A B E F.

For D C, F E are each equal to A B ——— ——— 89
Th. D C is equal to F E ——— ——— 48
Th. the line D F is equal to the line C E ——— 49
Ag. the ſide A D is equal to the ſide B C ——— 89
Th. A D, D F are equal to B C, C E each to each ——— ———
And the angle A D F is equal to the angle B C E ——— 81
Th. the triangle A D F is equal to the triangle B C E ——— 62
Th. the quadrangle A G C D is equal to B E F G ——— 50
Th. the parallelogram A B C D is equal to A B E F ——— 49

36 E. 1

92. **Parallelograms ſtanding upon equal baſes, and between the ſame parallels, are equal to one another.**

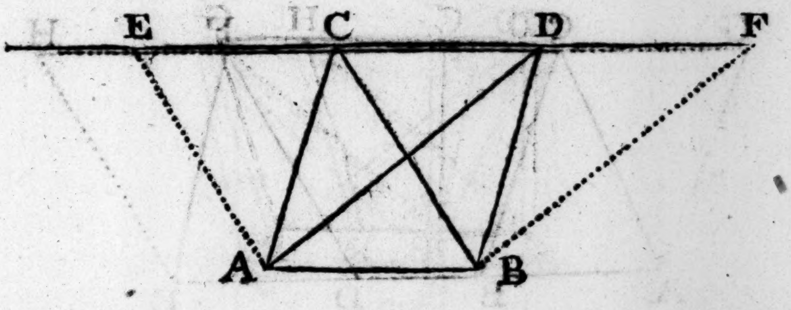


Let ABCD, EFGH be parallelograms ſtanding upon equal baſes AB, EF, and between the parallels AF, DG: then ABCD, EFGH are equal.

For join AH, BG.

Now AB is equal to EF ——— ——— theo.
And HG is equal to EF ——— ——— 89
Th. the lines AB, HG are equal ——— ——— 48
Th. the lines AH, BG are parallels ——— ——— 88
Th. ABGH is a parallelogram ——— ——— 22
Th. ABGH is equal to ABCD ——— ——— 91
Alſo ABGH is equal to EFGH ——— ——— 91
Th. the parallelograms ABCD, EFGH are equal ——— 48

93. **Triangles ſtanding upon the ſame baſe, and between the ſame parallels, are equal to one another.** 37 l.1.



Let ABC, ABD be triangles ſtanding upon the baſe AB, and between the parallels AB, CD: then the triangles ABC, ABD are equal.

For produce CD both ways to E and F, and draw AE, BF parallel to BC, AD —— —— —— —— p. 10

Now ABCE, ABFD are parallelograms —— 22

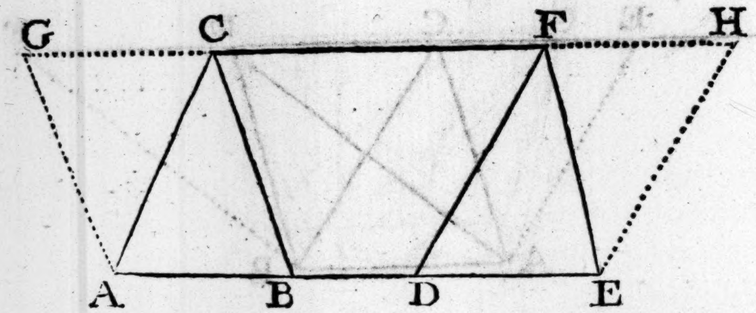
And ABCE is equal to ABFD —— —— 91

But the triangle ABC is half ABCE —— —— 89

And the triangle ABD is half ABFD —— —— 89

Th. the triangle ABC is equal to ABD —— 51

38 E. I. 94. Triangles ſtanding upon equal baſes, and between the ſame parallels, are equal to one another.



Let ABC, DEF be triangles ſtanding upon equal baſes AB, DE, and between the parallels AE, CF: then the triangles ABC, DEF are equal.

For produce CF both ways to G and H, and draw AG, EH parallel to BC, DF p. 10

Now ABCG, DEHF are parallelograms 22

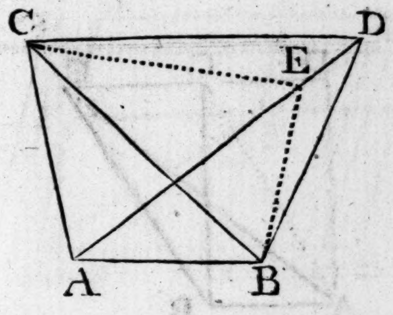
And ABCG is equal to DEHF 92

But the triangle ABC is half ABCG 89

And the triangle DEF is half DEHF 89

Th. the triangle ABC is equal to DEF 51

95. Equal triangles ſtanding upon the ſame baſe, and on the ſame ſide, are between the ſame parallels. 39. l. 1.



Let ABC, ABD be equal triangles ſtanding upon the baſe AB, and join CD: then the line CD is parallel to AB.

For if not, draw CE parallel to AB, and join BE.

Now the triangle ABC is equal to ABE —— 93

And the triangle ABC is equal to ABD —— theo.

Th. the triangles ABE, ABD are equal —— 48

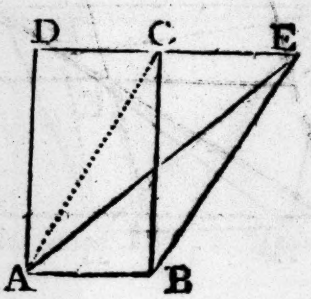
Which is impoſſible —— —— —— 52

Th. the line CE is not parallel to AB —— ——

Conſequently, CD muſt be parallel to AB —— ——

96. If

41. E. 1. 96. If a parallelogram and a triangle have the ſame baſe, and are between the ſame parallels, the parallelogram will be double to the triangle.



Let the parallelogram A B C D, and the triangle A B E be conſtituted upon the baſe A B, and between the parallels A B, D E: then the parallelogram A B C D is double to the triangle A B E.

For join A C.

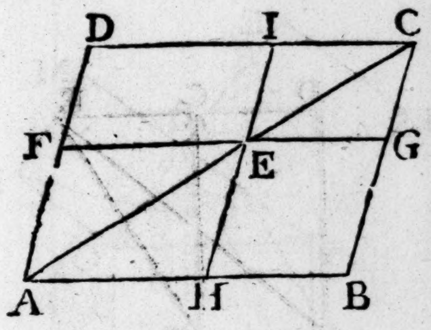
Now the triangles A B C, A B E are equal. —— 93

And the triangle A B C is half A B C D —— —— 89

Th. the triangle A B E is equal to half A B C D —— 56

Th. A B C D is double to the triangle A B E —— ——

97. The complements of a parallelogram are equal. 43 E.1.

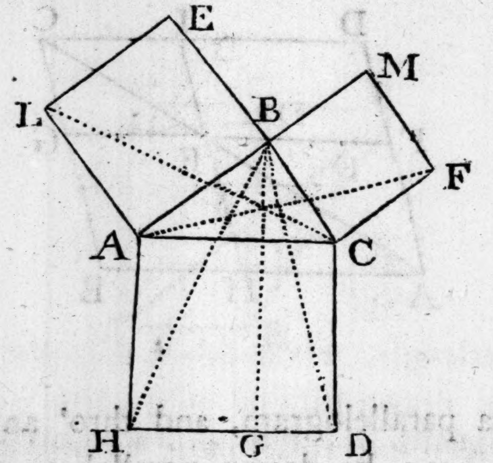


Let A B C D be a parallelogram, and thro' any point E in its diagonal A C, let F G, H I be drawn parallel to the ſides: then the complements H B G E, E I D F are equal.

For A H E F, E G C I are parallelograms —— —— 22
Th. the triangle A H E ——— = A E F —— —— 89
And the triangle E G C ——— = E C I —— —— 89
Th. the triangle A H E + E G C = A E F + E C I —— 49
Ag. the triangle A B C ——— = A C D —— —— 89
Th. the complem. H B G E ——— = E I D F —— 50

98. In

47 E.1 98. In right angled triangles, the ſquare of the ſide oppoſite to the right angle, is equal to both the ſquares of the ſides containing it.

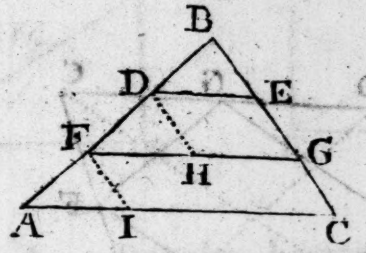


If the triangle ABC has a right angle at B, then $AC^2 = AB^2 + BC^2$.

For let AD, AE, BF be ſquares deſcribed upon AC, AB, BC by problem 11, draw BG parallel to AH or CD by problem 10, and join AF, CL, BD, BH.

Now the angle LAB is equal to CAH —— —— 54
Th. the angle LAC is equal to BAH —— —— 49
And LA, AC are equal to BA, AH each to each —— 24
Th. the triangle LAC is equal to the triangle BAH — 62
Again, EBC is a ſtrait line —— —— 68
Th. the ſquare AE is double to the triangle LAC —— 96
Alſo the parallelogram AG is double to the triangle BAH. 96
Th. the parallelogram AG is equal to the ſquare AE — 51
By the ſame reaſon GC is equal to CM —— ——
Th. $AG + GC = AE + CM$. —— —— —— 49
Or —— $AC^2 = AB^2 + BC^2$ —— —— 61

99. If lines drawn parallel to the baſe of a triangle divide one of its ſides into equal parts, they will alſo divide the other ſide into equal parts.



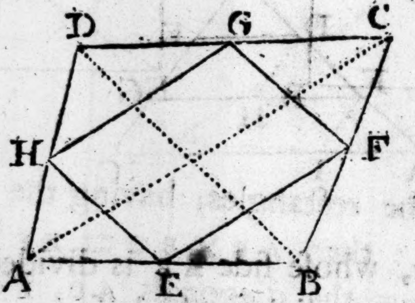
Let A B C be a triangle, whoſe ſide A B is divided into equal parts by the lines D E, F G drawn parallel to the baſe A C; then the ſide B C will likewiſe be divided into equal parts by the ſaid lines.

For let D H, F I be drawn parallel to the ſide B C — p. 10
Now the angles D B E, F D H, A F I are equal — 81
Alſo the angles B D E, D F H, F A I are equal — 81
And the ſides B D, D F, F A are equal — theo.
Th. the ſides B E, D H, F I are equal — 76
But E G, G C are equal to D H, F I — 89
Th. the parts B E, E G, G C are equal — 48

100. Corollary.

A right line that biſects two ſides of a triangle, muſt be parallel to the remaining ſide.

101. If the middle points of the ſides of any quadrangle be joined by four lines not cutting one another, the ſaid lines will form a parallelogram.

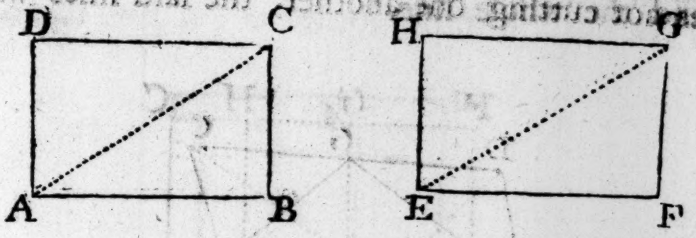


Let A B C D be a quadrangle whoſe ſides are biſected in E, F, G, H, and join E F, F G, G H, H E; then the figure E F G H is a parallelogram.

For draw the diagonals A C, B D.

Now the ſides A B, B C are biſected in E and F —— theo.
Th. E F is parallel to A C —— —— 100
By the ſame reaſon H G is parallel to A C —— ——
Th. the lines E F, H G are parallel —— —— 83
By the ſame reaſon E H, F G are parallel —— ——
Th. E F G H is a parallelogram —— —— 22

102. **The rectangles contained by equal lines, are equal to one another.**



Let ABCD, EFGH be rectangles, having the ſides AB, BC equal to EF, FG each to each: then AB × BC = EF × FG.

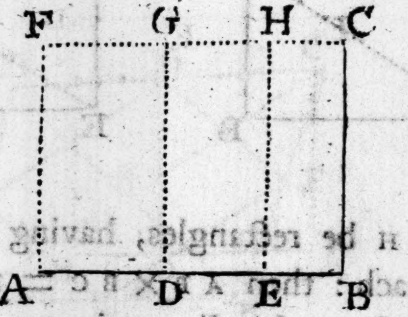
For draw the diagonals AC, EG.

Now the ſides AB, BC are equal to EF, FG each to each. theo.
And the angle ABC is equal to EFG —— —— 54
Th. the triangles ABC, EFG are equal —— —— 62
By the ſame reaſon, the triangles ADC, EHG are equal ——
Th. ABCD is equal to EFGH —— —— 49
Or AB × BC = EF × FG —— —— 61

103. Corollary.

The ſquares of equal lines are equal.

1.E.2. 104. The rectangle contained by a whole and a divided line, is equal to all the rectangles contained by the ſeveral parts of the divided line and the whole line.

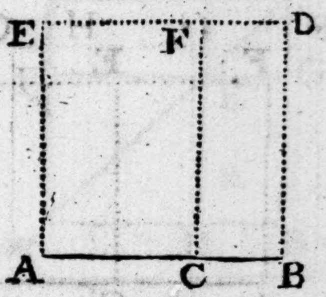


Let AB, BC be ſtrait lines, and divide AB at pleaſure in D and E: I ſay, $AB \times BC = AD \times BC + DE \times BC + EB \times BC$.

For erect BC at right angles to AB by problem 6, draw CF parallel to BA, and AF, DG, EH parallel to BC by problem 10.

Now ABC is a right angle —— con.
Th. AC, AG, GE, EC are rectangles —— 90
And AC is equal to $AG + GE + EC$ —— 52
Or $AB \times BC = AD \times DG + DE \times EH + EB \times BC$ —— 61
But BC, DG, EH are equal —— 89
Th. $AB \times BC = AD \times BC + DE \times BC + EB \times BC$ —— 56

105. If a line is any how divided, the rectangles contained by each of the parts and the whole line, will be equal to the ſquare of the ſaid line. 28.2.



Let the line AB be divided at pleaſure in C: then $AC \times AB + CB \times AB = AB^2$,

For upon AB deſcribe the ſquare $ABDE$ by problem 11, and draw CF parallel to AE or BD ——— ——— p. 10

Now AF, FB are rectangles ——— ——— 90

And $AF + FB$ is equal to AD ——— ——— 52

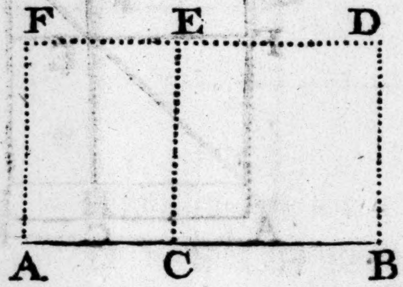
Or $AC \times AE + CB \times BD = AB^2$ ——— ——— 61

But AE, AB, BD are equal ——— ——— 24

Th. $AC \times AB + CB \times AB = AB^2$ ——— ——— 56

106. If

3E2. 106. If a line is divided into any two parts, the rectangle contained by the whole line and either part, will be equal to the rectangle contained by the parts with the ſquare of the part taken.



Let the line AB be any how divided in C, and take the part CB: then $AB \times CB = AC \times CB + CB^2$.

For upon CB deſcribe the ſquare CBDE, produce DE to F, and draw AF parallel to CE or BD.

Now AD, AE are rectangles —— —— 90

And AD is equal to AE + EB —— —— 52

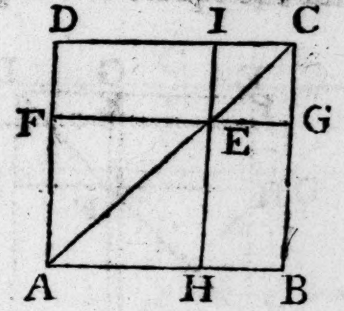
Or $AB \times BD = AC \times CE + CB \times BD$ —— —— 61

But CE, CB, BD are equal —— —— 24

Th. $AB \times CB = AC \times CB + CB^2$ —— —— 56

167. The

107. The parallelograms formed about the diagonal of a ſquare by lines drawn parallel to its ſides, are likewiſe ſquares. 42. 2.

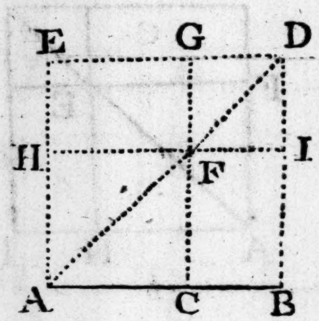


Let A B C D be a ſquare, and through any point E in the diagonal A C, draw F G, H I parallel to A B, B C: then F H, I G are the ſquares of A H, H B.

For the angle A C B is equal to B A C —— 63
And the angle A C B is equal to A E H —— 81
Th. the angles B A C, A E H are equal —— 48
Th. the ſides A H, H E are equal —— —— 64
But the ſides E F, F A are equal to A H, H E —— 89
Th. A H, H E, E F, F A are equal —— —— 48
Again, the angles at A, H, E, F are right ones, for they are equal to the angles of the ſquare A B C D —— 89
Th. A H E F is the ſquare of A H —— —— 24
And E G C I is the ſquare of I C or H B, by the ſame reaſon ——

108. If

4E.2. 108. If a line is divided into any two parts, the ſquare of the whole line, will be equal to the ſquares of the parts with twice a rectangle contained by the parts.



Let the line AB be divided at pleaſure in C: I ſay, $AB^2 = AC^2 + CB^2 + 2AC \times CB$.

For upon AB deſcribe the ſquare ABDE, join AD, and draw CFG, HFI parallel to AE, AB.

Now $ABDE = HC + GI + EF + FB$ ——— 52

And — $EF = FB$ ——— 97

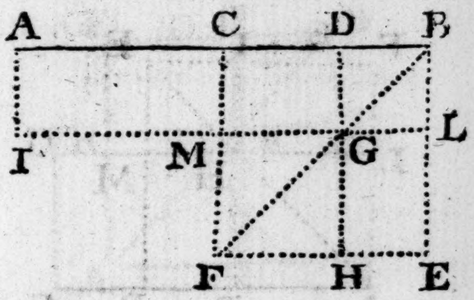
Th. $ABDE = HC + GI + 2EF$ ——— 56

Or — $AB^2 = HC + GI + 2HF \times FG$ ——— 61

But HC, GI are the ſquares of AC, CB ——— 107

Th. — $AB^2 = AC^2 + CB^2 + 2AC \times CB$ ——— 56

109. If a line is equally and unequally divided, the rectangle contained by the unequal parts with the ſquare of the intermediate diſtance, will be equal to the ſquare of half the line. 5l. 1.



Let the line AB be divided equally at C, and any how at D: I ſay, $AD \times DB + CD^2 = CB^2$.

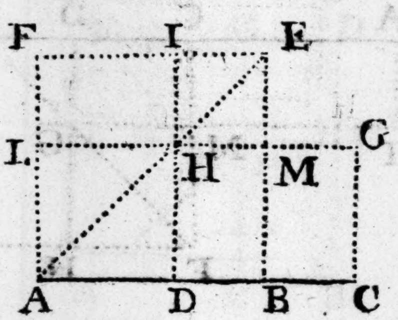
For upon CB deſcribe the ſquare CBEF, join BF, draw DGH, AI parallel to BE, and IGL parallel to AB.

Now CG ——— = GE —— —— 97
Th. CL ——— = DE —— —— 49
But AM ——— = CL —— —— 92
Th. AM ——— = DE —— —— 48
Th. AG ——— = DE + CG —— 49
Th. AG + MH —— = DE + CG + MH —— 49
Or $AD \times DG + MH = CB^2$ —— —— 61
But MH, DL are the ſquares of CD, DB —— 107
Th. $AD \times DB + CD^2 = CB^2$ —— —— 56

109. Other-

109*. Otherwiſe,

bE. 2. The rectangle contained by the ſum and difference of two lines, is equal to the difference of their ſquares.



Let AB, BC be unequal lines in the ſame direction, take BD equal to BC, then AC is their ſum, and AD is their difference: I ſay, $AC \times AD = AB^2 - BC^2$.

For upon AB deſcribe the ſquare ABEF, join AE, draw CG, DHI parallel to BE, and LHG parallel to AC.

Now BG —— $= BH$ —— —— 92

And FH —— $= BH$ —— —— 97

Th. BG —— $= FH$ —— —— 48

Th. $BG + LB = FH + LB$ —— —— 49

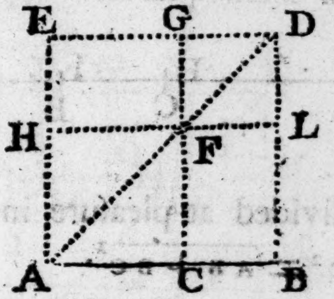
That is AG --- $= BF - IM$ —— —— ——

Or $AC \times AL = AB^2 - IM$ —— —— 61

But LD, IM are the ſquares of AD, DB, or BC —— 107

Th. $AC \times AD = AB^2 - BC^2$ —— —— 56

110. If a line is divided into any two parts, the ſquares of the whole line and either part, will be equal to twice a rectangle contained by the ſaid line and part taken with the ſquare of the other part. 7 l.s.



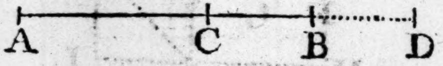
Let the line AB be divided at pleaſure in C, and take the part BC: I ſay, $AB^2 + BC^2 = 2AB \times BC + AC^2$.

For upon AB deſcribe the ſquare $ABDE$, join AD, draw CFG parallel to AE, and HFL parallel to AB.

Now EF ——— $= FB$ ——— ——— 97
Th. EB ——— $= 2EF + GL + HC$ ——— 52
Th. $EB + GL = 2EF + 2GL + HC$ ——— 49
Th. $EB + GL = 2EL + HC$ ——— ——— 56
Or $AB^2 + GL = 2ED \times DL + HC$ ——— 61
But GL, HC are the ſquares of BC, CA ——— 107
Th. $AB^2 + BC^2 = 2AB \times BC + AC^2$ ——— 56

111. If

881. **111.** If a line is divided into any two parts, four times the rectangle contained by the whole line and either part with the ſquare of the other part, will be equal to the ſquare of the ſum of the ſaid line and part taken.



Let the line AB be divided at pleaſure in C, and take the part BC: I ſay, $4AB \times BC + AC^2 = \overline{AB + BC}^2$.

For produce AB, and make BD equal to BC.

Now $AB^2 + BD^2 + 2AB \times BD$ —— $= AD^2$ —— 108

Th. $AB^2 + BC^2 + 2AB \times BC$ —— $= AD^2$ —— 56

Ag. $2AB \times BC + AC^2$ —— $= AB^2 + BC^2$ —— 110

Th. $2AB \times BC + AC^2 + 2AB \times BC = AD^2$ —— 56

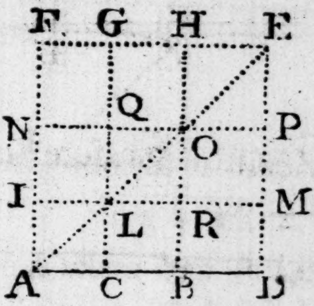
Or $4AB \times BC + AC^2$ —— $= AD^2$ —— 56

But $AB + BC$ —— $= AD$ —— con.

Th. $4AB \times BC + AC^2$ —— $= \overline{AB + BC}^2$ —— 56

111.* Otherwiſe,

If a line is biſected and another be added to it, the ſquare of the ſum, will be equal to four times a rectangle contained by the half line and remainder, with the ſquare of the added line.



Let the line A B be equally divided in C, and continued to D: then $AD^2 = 4\,AC \times CD + BD^2$.

For upon A D deſcribe the ſquare A D E F, join A E, draw C G, B H parallel to A F, and I L M, N O P parallel to A D.

Now N B, I C, Q R, H P are the ſquares of A B, A C, C B, B D. 107

And A C, C B are equal —— —— —— theo.

Th. I C, Q R, B L, L N, are equal —— —— 103

Ag. $NG = GO$, and $OM = MB$ —— —— 92

But G O is equal to O M —— —— 97

Th. N G, G O, O M, M B are equal —— —— 48

Th. I G, G R, L D, and $IC + OM$ are equal —— 49

Th. $FO + ND = 4\,LD$ —— —— 49

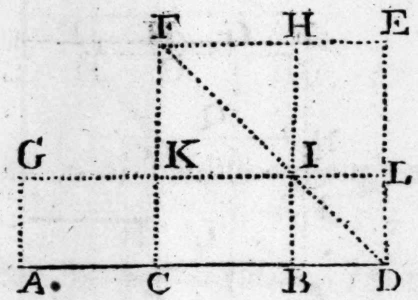
Th. $FO + ND + HP = 4\,LD + HP$ —— 49

Or $AD^2 = 4\,CL \times CD + BD^2$ —— —— 61

Th. $AD^2 = 4\,AC \times CD + BD^2$ —— —— 56

112. If

112. If a line is divided equally, and another be added to it, the rectangle contained by the ſum and added line, with the ſquare of half the line will be equal to the ſquare of the line compoſed of the half and added line.



If the line AB is divided equally in C, and produced to D; then $AD \times DB + CB^2 = CD^2$.

For let CDEF be the ſquare of CD, join DF, draw AG, BH parallel to CF, and thro' I draw GL parallel to AD.

Now AK ———————— $= CI$ ———— ———— 92

And IE ———————— $= CI$ ———— ———— 97

Th. AK ———————— $= IE$ ——— ———— 48

Th. $AK + CL$ ———————— $= IE + CL$ ———— 49

Or AL ———————— $= CL + IE$ ———— ———

Th. $AL + KH$ ———————— $= CL + IE + KH$ ——— 49

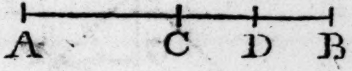
Or $AD \times DL + KH$ ---- $= CD^2$ ———— 61

But KH, BL are ſquares of CB, BD ———— 107

Th. $AD \times DB + CB^2$ ---- $= CD^2$ ——— 56

Same as the 109. Sel. Dem. 12 Bk 2.

113. If a line is equally and unequally divided, the ſquares of the unequal parts, will be double to the ſquares of the half line and intercepted part. 9 l. 2.



Let the line AB be equally divided in C, and any how in D: Then

$AD^2 + DB^2 = 2AC^2 + 2CD^2$ —— —— ——

For AD^2 —— $= AC^2 + CD^2 + 2AC \times CD$ —— 108

Th. $AD^2 + DB^2 = AC^2 + CD^2 + 2AC \times CD + DB^2$ — 49

But BC —— $= AC$ —— —— theo.

Th. $AD^2 + DB^2 = BC^2 + CD^2 + 2BC \times CD + DB^2$ — 56

Ag. $BC^2 + CD^2 =$ —— —— $2BC \times CD + DB^2$ — 110

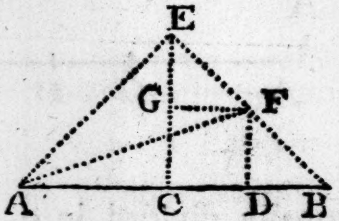
Th. $AD^2 + DB^2 = BC^2 + CD^2 + BC^2 + CD^2$ —— 56

Or $AD^2 + DB^2 = 2BC^2 + 2CD^2$ —— 52

Th. $AD^2 + DB^2 = 2AC^2 + 2CD^2$ —— 56

113.* Otherwiſe,

If the line AB is equally divided in C, and at pleaſure in D: I ſay, $AD^2 + DB^2 = 2AC^2 + 2CD^2$.



For erect CE at right angles to AB, and make it equal to AC, join AE, EB, draw DF, FG parallel to CE, BA, and join AF.

Now the angle $AEC = CAE$, and $EBC = CEB$ —— 63

Th. each is half a right angle —— —— 86

Th. AEB is a right angle —— —— 52

Alſo EGF, FDB are right angles, for each is equal to the right angle ECB —— —— —— 81

And the angles EFG, DFB are half right angles, being equal to the angles EBC, CEB —— —— 81

Th. EG is equal to GF, and FD is equal to DB —— 64

Again, CD is equal to GF —— —— 89

Th. $AC^2 = CE^2$, $EG^2 = GF^2 = CD^2$, and $FD^2 = DB^2$. 103

Th. $AE^2 = 2AC^2$, $EF^2 = 2GF^2$, and $AF^2 = AE^2 + EF^2$. 98

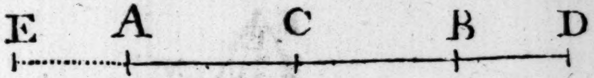
Th. $AF^2 = 2AC^2 + 2GF^2$ —— —— 56

But $AF^2 = AD^2 + DF^2$ —— —— 98

Th. $AD^2 + DF^2 = 2AC^2 + 2GF^2$ —— —— 48

Th. $AD^2 + DB^2 = 2AC^2 + 2CD^2$ —— —— 56

114. If a line is equally divided and another be added to it, the ſquares of the ſum and the added line, will be double to the ſquares of half the line, and the line compoſed of the half and added line. 102. 2.

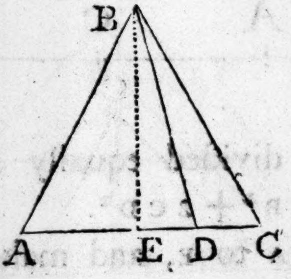


Let the line AB be divided equally in C, and continued to D; then $AD^2 + BD^2 = 2CB^2 + 2CD^2$.

For produce DA to E, and make AE equal to BD.

Now EA is equal to BD ——— ——— con.
And AC is equal to CB ——— ——— theo.
Th. EC is equal to CD ——— ——— 49
And EB is equal to AD ——— ——— 49
Therefore, the line ED is equally divided in C, and unequally in B ——— ——— ——— ———
Th. $EB^2 + BD^2 = 2CB^2 + 2CD^2$ ——— 113
But the ſquares of EB and AD are equal ——— 103
Th. $AD^2 + BD^2 = 2CB^2 + 2CD^2$. ——— 56

115. In Iſoſceles triangles, the ſquare of the ſide, is equal to the ſquare of a line joining the vertex and any point in the baſe, with a rectangle contained by the parts of the baſe.

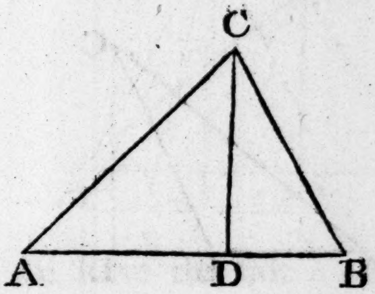


Let the triangle A B C have the ſide A B equal to the ſide B C, and join the vertex to any point D in the baſe, then $AB^2 = BD^2 + AD \times DC$.

For divide A C equally in E by problem 5, and join B E.

Now the angles A E B, B E C are equal ——— 65
Th. A E B, B E C are right angles ——— ——— 9
Th. $AB^2 = BE^2 + EA^2$ ——— ——— 98
But $AD \times DC + ED^2 = EA^2$ ——— ——— 109
Th. $AB^2 = BE^2 + AD \times DC + ED^2$ ——— 56
Or $AB^2 = BE^2 + ED^2 + AD \times DC$ ——— 52
Ag. $BD^2 = BE^2 + ED^2$ ——— ——— 98
Th. $AB^2 = BD^2 + AD \times DC$ ——— ——— 56

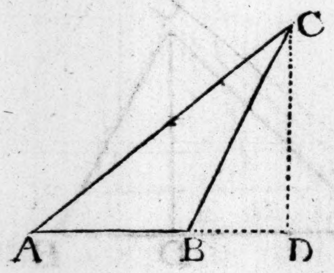
116. In triangles, the ſquares of the baſe and either ſide, are equal 132. 2
to twice a rectangle contained by the baſe and adjacent ſegment made by a perpendicular, with the ſquare of the other ſide.



Let the triangle ABC, have a perpendicular CD falling from the vertex to the baſe, and take the ſide BC: Then $AB^2 + BC^2 = 2AB \times BD + AC^2$.

For $AB^2 + BD^2$ —— $= 2AB \times BD + AD^2$ —— 110
Th. $AB^2 + BD^2 + DC^2 = 2AB \times BD + AD^2 + DC^2$ —— 49
But BC^2 ———— $= BD^2 + DC^2$ —— —— 98
Th. $AB^2 + BC^2$ —— $= 2AB \times BD + AD^2 + DC^2$ —— 56
Ag. AC^2 ———— $=$ ———— $AD^2 + DC^2$ —— 98
Th. $AB^2 + BC^2$ —— $= 2AB \times BD + AC^2$ —— 56

12 l. 2. 117. In obtuſe angled triangles, the ſquare of the ſide oppoſite the obtuſe angle, is equal to the ſquares of the ſides about it, with twice a rectangle contained by one of thoſe ſides, and its continuation to a perpendicular falling thereon from the extremity of the other ſide.



Let the triangle ABC have an obtuſe angle at B, and let CD be perpendicular to the continuation of AB; then $AC^2 = AB^2 + BC^2 + 2\,AB \times BD$.

For $AC^2 = AD^2 + DC^2$ — — — 98

And $AD^2 = AB^2 + BD^2 + 2\,AB \times BD$ — 108

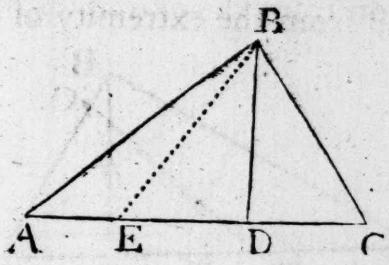
Th. $AC^2 = AB^2 + BD^2 + 2\,AB \times BD + DC^2$ — 56

Or $AC^2 = AB^2 + BD^2 + DC^2 + 2\,AB \times BD$ — 52

But $BC^3 =$ — $BD^2 + DC^2$ — — 98

Th. $AC^2 = AB^2 + BC^3 + 2\,AB \times BD$ — 56

118. The difference of the ſquares of two ſides of a triangle, is equal to the difference of the ſquares of the ſegments of the baſe made by a perpendicular.



Let the triangle ABC have the ſide AB greater than the ſide BC, and let BD be perpendicular to the baſe AC: then $AB^2 - BC^2 = AD^2 - DC^2$.

For AB^2 —— $= AD^2 + DB^2$ ——— —— 98
And BC^2 —— $= DC^2 + DB^2$ ——— —— 98
Th. $AB^2 - BC^2 = AD^2 - DC^2$ ——— —— 50

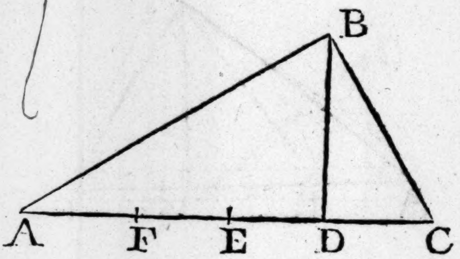
119. Corollary.

The difference of the ſquares of two ſides of a triangle, is equal to a rectangle contained by the baſe and difference of the ſegments made by the perpendicular.

For make DE equal to DC.

Now $AC = AD + DC$, and $AE = AD - DC$ are the ſum and difference of the ſegments, becauſe DE is equal to DC —— con.
Th. $AC \times AE = AD^2 - DC^2$ ——— ——— 109*
Th. $AB^2 - BC^2 = AC \times AE$ ——— ——— 48

120. The rectangle contained by the ſum and difference of the ſides of a triangle, will be double the rectangle contained by the baſe, and diſtance of the perpendicular from the middle of the baſe.



Let ABC be a triangle with a perpendicular BD falling upon the baſe AC, whoſe middle point is E.

Then $\overline{AB+BC} \times \overline{AB-BC} = 2AC \times ED$. For make AF equal to DC, whence $AD - DC = FD$. 50

Now $\overline{AB+BC} \times \overline{AB-BC} = AB^2 - BC^2$ 109*

And $AC \times FD = AD^2 - DC^2$ 109*

But $AB^2 - BC^2 = AD^2 - DC^2$ 118

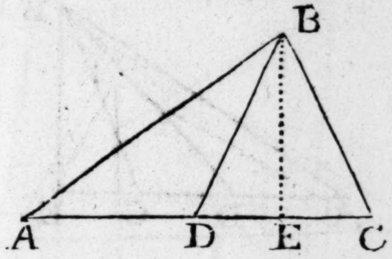
Th. $\overline{AB+BC} \times \overline{AB-BC} = AC \times FD$ 48

Ag. $FE = ED$ 50

Th. $FD = 2ED$ 49

Th. $\overline{AB+BC} \times \overline{AB-BC} = 2AC \times ED$ 56

121. If the baſe of a triangle is biſected by a line drawn from the vertex, the ſquares of the ſides will be double to the ſquares of the ſaid line and half the baſe.



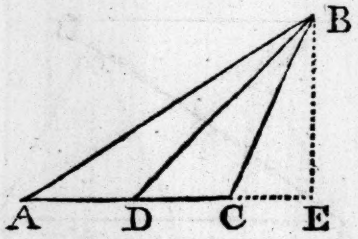
Let the triangle A B C, have its baſe A C equally divided by the line B D: then $AB^2 + BC^2 = 2BD^2 + 2DC^2$.

For upon A C let fall the perpendicular B E ——— p. 7
Now AB^2 —— $= AE^2 + EB^2$ —— —— 98
And BC^2 —— $= EC^2 + EB^2$ —— —— 98
Th. $AB^2 + BC^2 = AE^2 + EC^2 + 2EB^2$ ——— 49
But $AE^2 + EC^2 = 2DC^2 + 2DE^2$ —— —— 113
Th. $AB^2 + BC^2 = 2DC^2 + 2DE^2 + 2EB^2$ ——— 56
Ag. BD^2 —— $=$ ——— $DE^2 + EB^2$ ——— 98
Th. $2BD^2$ —— $=$ ——— $2DE^2 + 2EB^2$ —— 49
Th. $AB^2 + BC^2 = 2DC^2 + 2BD^2$ —— —— 56
Or $AB^2 + BC^2 = 2BD + 2DC^2$ —— —— ——

Caſe

Caſe 2.

When the triangle ABC has an obtuſe angle at the baſe, I ſay as before, $AB^2 + BC^2 = 2BD^2 + 2DC^2$.



For let BCA be the obtuſe angle, produce AC to E, and let BE fall at right angles to AE. —— —— —— p. 7

Now AB^2 ——— $= AE^2 + EB^2$ —— —— 98

And BC^2 ——— $= CE^2 + EB^2$ —— —— 98

Th. $AB^2 + BC^2 = AE^2 + CE^2 + 2EB^2$ ——— 49

But $AE^2 + CE^2 = 2DC^2 + 2DE^2$ —— —— 114

Th. $AB^2 + BC^2 = 2DC^2 + 2DE^2 + 2EB^2$ —— 56

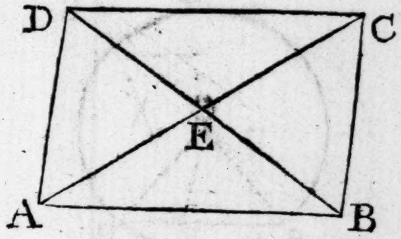
Ag. BD^2 ——— $=$ ——— $DE^2 + EB^2$ —— 98

Th. $2BD^2$ ——— $=$ ——— $2DE^2 + 2EB^2$ —— 49

Th. $AB^2 + BC^2 = 2DC^2 + 2BD^2$ —— —— 56

Or $AB^2 + BC^2 = 2BD^2 + 2DC^2$.

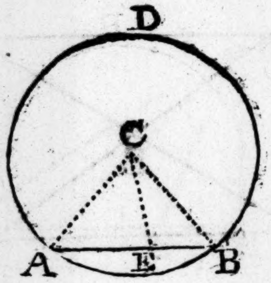
122. In parallelograms, the diagonals cut each other into equal parts, and their ſquares both together, are equal to the ſquares of the four ſides taken together.



Let ABCD be a parallelogram whoſe diagonals cut each other in E: then AE is equal to EC, BE is equal to ED, and $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$.

For the angles ABE, EAB are equal to CDE, ECD each to each. 80
And the correſponding ſides AB, CD are equal —— 89
Th. AE is equal to EC, and BE is equal to ED —— 76
Ag. $2AE^2 + 2EB^2 = AB^2 + BC^2$ —— —— 121
Alſo $2AE^2 + 2ED^2 = CD^2 + DA^2$ —— —— 121
And —— $ED^2 = EB^2$ —— —— 103
Th. $2AE^2 + 2EB^2 = CD^2 + DA^2$ —— —— 56
Th. $4AE^2 + 4EB^2 = AB^2 + BC^2 + CD^2 + DA^2$ —— 49
But $4AE^2$ —— $= AC^2$, and $4EB^2 = BD^2$ —— 108
Th. $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$ —— 56

263 123. A ſtrait line that joins any two points in the circumference of a circle, will fall within the circle.

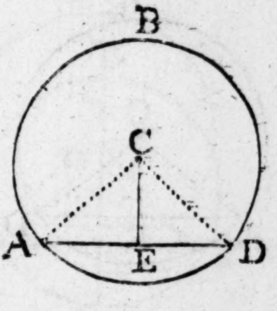


Let A and B be points in the circumference of a circle A D B, and join A B; then the line A B falls within the circle.

For find C the center of the circle by problem 32, in A B aſſume the point E, and join C E, C A, C B.

Now the ſide C A is equal to the ſide C B 30
Th. the angle C B A is equal to C A B ~~60~~ 63
But the angle C E B is greater than C A B 70
Th. the angle C E B is greater than C B A 56
Th. the line C B is greater than the line C E 72
Th. the point E is within the circle 30
Th. the line A B muſt fall within the circle

124. A line drawn from the center of a circle, to the middle of 3E.3. a line terminated by the circumference, will meet the ſame at right angles.

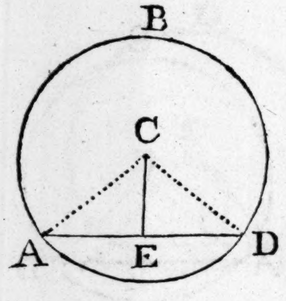


In a circle ABD, whoſe center is C, let the line AD be equally divided by the line CE: then AEC, CED are right angles.

For join AC, CD.

Now AE is equal to ED ——— ——— theo.
.... AC is equal to CD ——— ——— 30
And CE is common ——— ——— ———
Th. EA, AC, CE are equal to ED, DC, CE each to each.
Th. the angles AEC, CED are equal ——— 65
Th. AEC, CED are right angles ——— ——— 9

383 125. A line drawn from the center of a circle, at right angles, to a line terminated by the circumference; will divide it equally.

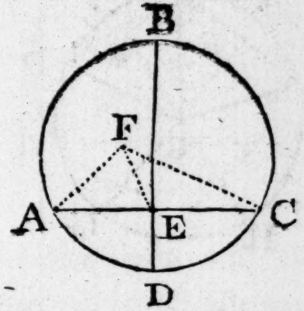


In the circle A B D, whoſe center is C, let the line C E be drawn at right angles to the line A D: then the ſegments A E, E D are equal.

For join A C, C D.

Now C D is equal to C A ——— ——— 30
Th. the angles D A C, C D A are equal ——— 63
Alſo the angles A E C, C E D are equal ——— 54
Th. the angles E A C, C E A are equal to E D C, C E D each to each ——— ——— ———
And the correſponding ſide C E is common.
Th. A E is equal to E D ——— ——— 76

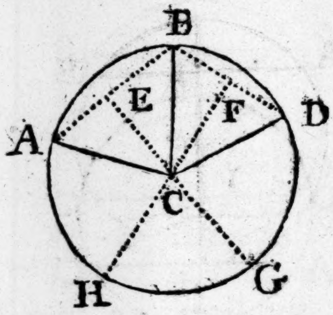
126. If a line terminated by a circle, is divided equally by a line cutting it at right angles, the cutting line will paſs thro' the center of the circle.



In the circle A B C D, let the line A C be equally divided in E, by the line B D cutting it at right angles: then B D will paſs thro' the center of the circle. For if not, let F be the center, and join F A, F E, F C.

Now A E is equal to E C — — theo.
----- F A is equal to F C — — 30
And E F is common — — —
Th. the angles A E F, F E C are equal — 65
Th. each is a right angle — — 9
Th. the angle A E F is equal to A E B — — 54
Which is impoſſible — — — 52
Th. B D muſt paſs thro' the center of the circle — —

9.8.3. **127. If from a point within a circle, three equal lines be drawn to the circumference, that point is the center of the circle.**

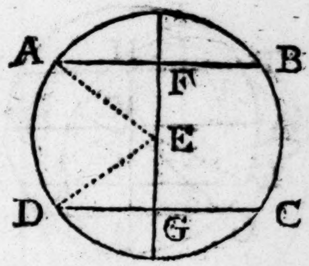


Let ABD be a circle in which three equal lines CA, CB, CD are drawn from the point C to the circumference: then C is the centre of the circle.

For join AB, BD, biſect them in E and F by problem 5, draw EC, FC and continue them to G and H.

Now CA is equal to CB —— —— theo.
----- AE is equal to EB —— —— —— con.
And CE is common —— —— —— ——
Th. the angles AEC, CEB are equal —— —— 65
Th. AEC, CEB are right angles —— —— 9
Th. the line ECG paſſes thro' the center —— 126
By the ſame reaſon, the line FCH paſſes thro' the center ——
Th. the point C muſt be the center of the circle. —— ——

128. **In a circle, lines equally diſtant from the center are equal to one another.** 14 l. 3.



In the circle ABCD, let the lines AB, DC be equally diſtant from the center E, namely, let the perpendiculars EF, EG falling upon them be equal: then AB, DC are equal.

For join AE, ED.

Now EA^2 —— $= AF^2 + FE^2$ —— —— 98

And ED^2 —— $= DG^2 + GE^2$ —— —— 98

But EA^2 —— $= ED^2$ —— —— 103

Th. $AF^2 + FE^2 = DG^2 + GE^2$ —— —— 48

Again, $FE^2 =$ —— GE^2 —— —— 103

Th. AF^2 —— $= DG^2$ —— —— —— 50

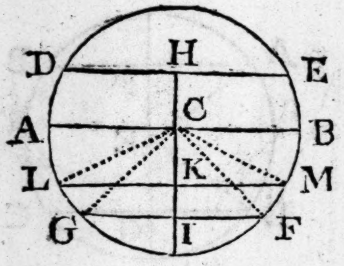
Th. AF —— $= DG$ —— —— —— 103

But AF —— $= FB$, and $DG = GC$ —— 125

Th. AF, FB, DG, GC are equal —— —— 48

Th. AB is equal to DC —— —— 49

15 E 3. **129. In circles, the greateſt line is a diameter, and of other lines the nearer to the center is the greater.**

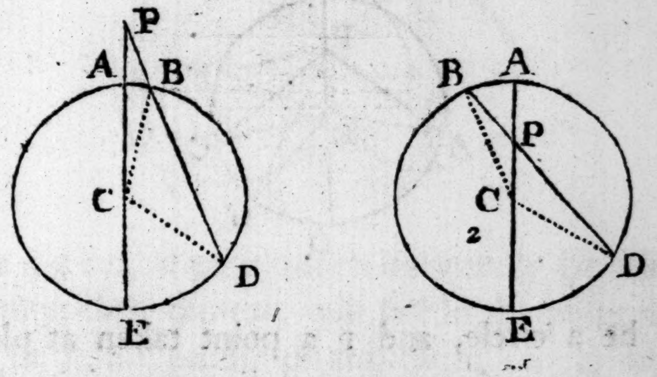


In the circle DEFG, whoſe center is C, and diameter AB, let the line DE be drawn nearer to the center than the line GF; then AB is greater than DE, and DE is greater than GF.

For upon DE, GF let fall the perpendiculars CH, CI, make CK equal to CH, thro' K draw LM at right angles to CI, and join CM, CF, CG, CL.

Now AC, CB, CM, CF, CG, CL are equal ——— 30
Th. AB is equal to LC + CM ——— ——— 49
But LC + CM is greater than LM ——— ——— 73
Th. AB is greater than LM ——— ——— 56
Again, DE is equal to LM ——— ——— 128
Th. AB is greater than DE ——— ——— 56
Secondly, the angle LCM is greater than GCF ——— 52
Th. LM is greater than GF ——— ——— 74
Th. DE is greater than GF ——— ——— 56

130. If from any point which is not the center, ſtrait lines be drawn to the circumference of a circle, the greateſt line will paſs thro' the center, and the leaſt will fall in the oppoſite part of the circumference. 8 l.3

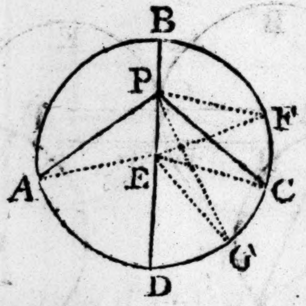


Let ABDE be a circle, C its center, P a point taken at pleaſure, and PD, PE lines drawn to the circumference, whereof PE paſſes thro' the center; then PE is greater than PD, and PB is greater than PA.

For join BC, CD.

Now CA, CB, CD, CE are equal 30
Th. PE is equal to PC + CD 49
But PC + CD is greater than PD 73
Th. PE is greater than PD 56
Secondly, PB + BC is greater than PC, or PA + AC 73
Th. ---- PB + AC is greater than PA + AC 56
Th. ----- PB is greater than PA 50
Again, PB + PC is greater than CB, fig. 2. 73
Th. PB + PC is greater than CA, or PA + PC 56
Th. PB is greater than PA 50

72.3 131. From an aſſumed point which is not the center of a circle, only two equal lines can be drawn to the circumference.



Let A B C D be a circle, and P a point taken at pleaſure which is not the center: then only two equal lines can join the point P and the circumference.

For find E the center of the circle, join P E, E A, make the angle P E C equal to the angle P E A, and draw A P, P C, C E.

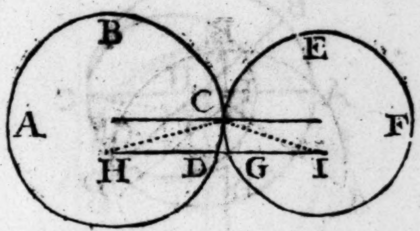
Now the line E A is equal to the line E C —— 30
Th. the ſides P E, E C are equal to P E, E A each to each —— ——
And the contained angles P E C, P E A are equal —— con.
Th. P C is equal to P A —— —— —— 62
Again, let the points F, G be aſſumed in the ſemicircle B C D, and join P F, F E, E G, G P.
Now the ſides P E, E G are equal to P E, E C each to each. above
And the contained angle P E G is greater than P E C —— 52
Th. P G is greater than P C —— —— —— 74
By the ſame reaſon P C is greater than P F. —— ——
Th. only P C is equal to P A.

132. Corollary. One circle can cut another, in two points only.

133. If

12l.3

133. If two circles touch one another, a ſtrait line joining their centers will fall in the point of contact.



If the circles A B C D, C E F G touch outwardly in the point C; then a ſtrait line joining their centers, will fall in the point of contact C.

For if not, let H the center of the circle A B C D and I the center of the circle C E F G, be joined by the ſtrait line H I cutting the circles in D and G, and join H C, C I.

Now H D is equal to H C — — 30

And G I is equal to C I — — 30

Therefore H D + G I is equal to H C + C I — 49

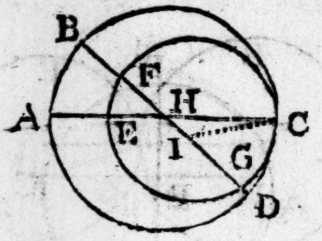
But H I is greater than H D + G I — — 52

Therefore H I is greater than H C + C I — 56

Which is impoſſible — — — 73

Therefore a ſtrait line joining the centers muſt fall in the point of contact.

118.3. 133. **Caſe 2. When the circles touch inwardly.**

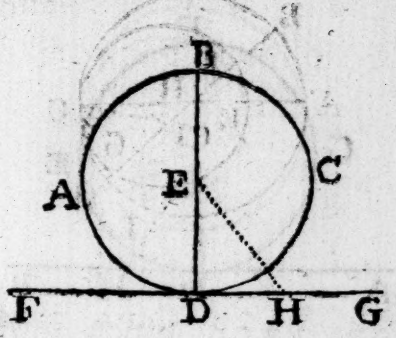


Let the circles ABCD, EFCG touch inwardly in the point C; then a ſtrait line joining their centers will fall in the point of contact C. For if not, let H the center of ABCD, and I the center of EFCG be joined by the line HD, cutting the circles in G and D, and join IC.

Now HI + IC is greater than HC 73
And HD is equal to HC 30
Th. HI + IC is greater than HD 56
Or HI + IC is greater than HI + ID
Th. —— IC is greater than —— ID 50
But IG is equal to IC 30
Th. IG is greater than ID 56
Which is abſurd 52
Therefore a ſtrait line joining the centers, muſt fall in the point of contact.

134. Cor. One circle can touch another, in one point only.

135. A ſtrait line drawn at right angles to the extremity of a diameter, is a tangent to the circle. 162 3.



Let ABCD be a circle whoſe center is E, and diameter BD, and thro' the point D let the line FG be drawn at right angles to BD; then the line FDG is a tangent to the circle.

For in FG aſſume the point H, and join EH.

Now EDH is a right angle ——— ——— theo.

Th. the angle EDH is equal to EHD + DEH ——— 86

Th. the angle EDH is greater than EHD ——— 52

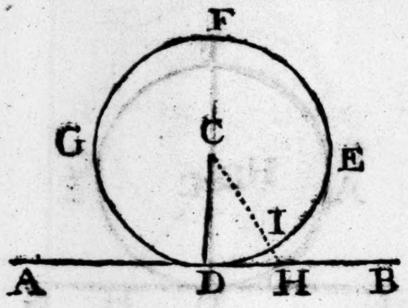
Th. the line EH is greater than the line ED ——— 72

Th. the point H is without the circle ——— ——— 30

Th. the line FDG is a tangent to the circle. ——— 38

136. Cor. A ſtrait line can touch a circle in one point only.

18 &13. 137. A line drawn from the center of a circle to the point of contact, will be perpendicular to the tangent.

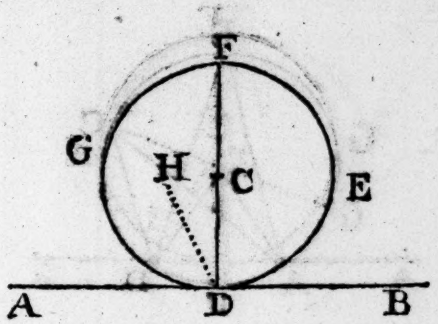


Let the line A B touch the circle D E F G in D, and from the center C draw C D: then the line C D is perpendicular to the tangent A B.

For if not, upon A B let fall the perpendicular C H by problem 7, cutting the circumference in I.

Now C H D is a right angle ——— ——— 9
Th. the angle C H D is equal to both the angles C D H, H C D. 86
Th. the angle C H D is greater than the angle C D H ——— 52
Th. the line C D is greater than the line C H ——— 72
But the lines C D, C I are equal ——— ——— 30
Th. C I is greater than C H ——— ——— 56
Which is abſurd. ——— ——— ——— 52
Th. C D muſt be perpendicular to A B ——— ——— —

138. A line drawn at right angles to a tangent, at the point of contact, will paſs thro' the center of the circle. 19.2.3.



Let the tangent A B touch the circle D E F G in D, and let D F be drawn at right angles to A B: then the line D F paſſes thro' the center of the circle C.

For if not, let H be the center, and join H D.

Now H D is perpendicular to A B ——— ——— 137

Th. A D H is a right angle ——— ——— 9

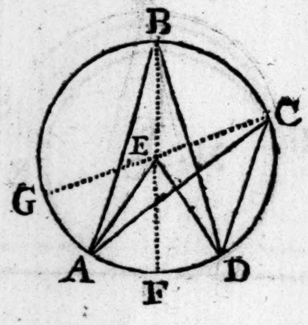
Alſo A D F is a right angle ——— ——— theo.

Th. the angles A D H, A D F are equal ——— 54

Which is impoſſible ——— ——— ——— 52

Therefore, the perpendicular D F muſt paſs thro' C the center of the circle.

20 l. 3. 139. An angle at the center of a circle is double to an angle at the circumference, ſtanding upon the ſame arch.



In the circle ABCD, let the angle AED at the center, and the angle ABD at the circumference, ſtand upon the arch AD; then the angle AED is double to the angle ABD.

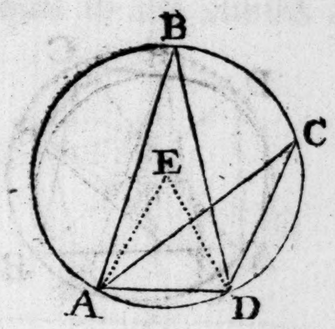
For join BE, and produce it to F.

Now EA is equal to EB ——— ——— 30
Th. the angle ABE is equal to EAB ——— ——— 63
Th. the angle ABE + EAB is double to ABE ——— 49
But the angle AEF is equal to ABE + EAB ——— 84
Th. the angle AEF is double to ABE ——— 56
Alſo, the angle FED is double to EBD, by the ſame reaſon. ———
Th. the angle AED is double to ABD ——— ——— 49

Again, let the angle ACD be taken lower, and produce the line CE to G.

Then the angle GED is double to GCD ——— above
And the angle GEA is double to GCA ——— above
Th. the angle AED is double to ACD ——— 50

140. All angles in the ſame ſegment of a circle, are equal to one another.

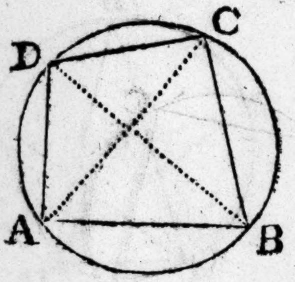


Let the circle ABCD be cut by the right line AD, and join AB, BD, AC, CD: then the angles ABD, ACD in the ſame ſegment are equal.

For find E the center of the circle by problem 32, and draw AE, ED.

Now the angle AED is double to ABD ——— 139
And the angle AED is double to ACD ——— 139
Th. each of the angles ABD, ACD is equal to half AED. ———
Th. the angles ABD, ACD are equal ——— ——— 51

22.E3. **141. The oppoſite angles of any quadrangle inſcribed in a circle, are equal to two right angles.**



Let a circle have a quadrangle ABCD inſcribed in it: then its oppoſite angles both together, are equal to two right angles.

For join AC, BD.

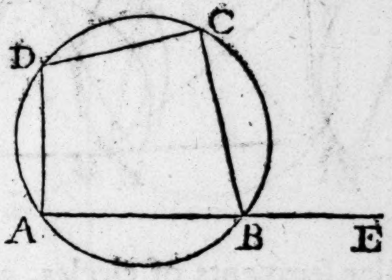
Now the angle ABD ——— = ACD ——— ——— 140
And the angle DBC ——— = DAC ——— ——— 140
Th. the angle ABC ——— = ACD + DAC ——— 49
Th. the angle ABC + CDA = ACD + DAC + CDA — 49
But two right angles ——— = ACD + DAC + CDA — 84
Th. the angle ABC + CDA = two right angles ——— 48

And by the ſame reaſon, the angle BAD + DCB is equal to two right angles.

142. Co-

142. Corollary.

If any ſide of a quadrangle inſcribed in a circle is produced, the outward angle will be equal to the inward and oppoſite angle.



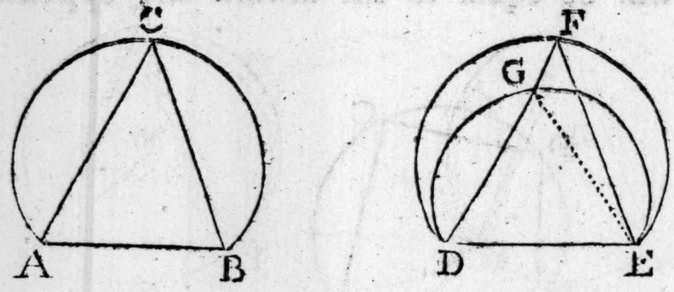
For let the ſide A B of a quadrangle A B C D inſcribed in a circle, be produced to E.

Now the angle A B C + C B E = two right angles —— 66
And the angle A B C + A D C = two right angles —— 141
Th. the angle A B C + C B E = A B C + A D C —— 48
Th. the angle —— C B E = A D C —— —— 50

143. Corollary 2.

If two oppoſite angles of a quadrangle taken together be equal to two right angles, a circle may be circumſcribed about the figure.

24ℓ3. **144. Similar ſegments of circles ſtanding upon equal baſes, are equal to one another.**



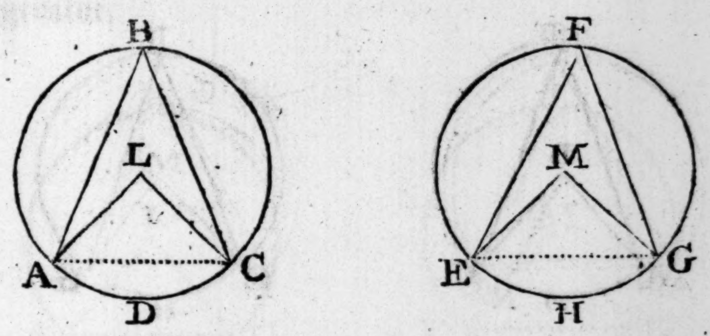
Let A B C, D E F be ſimilar ſegments of circles, conſtituted upon equal baſes A B, D E: then the ſegments A B C, D E F are equal.

For conceive the ſegment A B C applied to D E F with the point A on the point D, and the line A B on the line D E, then the point B will agree with E, becauſe A B is equal to D E ——— ——— theo.

Now if it be poſſible, let the ſegment A B C diſagree with the ſegment D E F, and fall into a different poſition D G E, and join A C, C B, D F, F E, E G.

Now the angle D G E is equal to A C B ——— 140
And the angle D F E is equal to A C B ——— 36
Th. the angle D G E is equal to D F E ——— ——— 48
Which is impoſſible ——— ——— ——— 70
Th. the ſegment A B C muſt agree with D E F ——— ———
Th. the ſegments A B C, D E F are equal ——— 53

145. In equal circles, equal angles either at the centers or circumferences, intercept equal arches. 26.2.3



In the equal circles ABCD, EFGH, let the angles ABC, EFG at the circumferences be equal: then the intercepted arches ADC, EHG, are alſo equal.

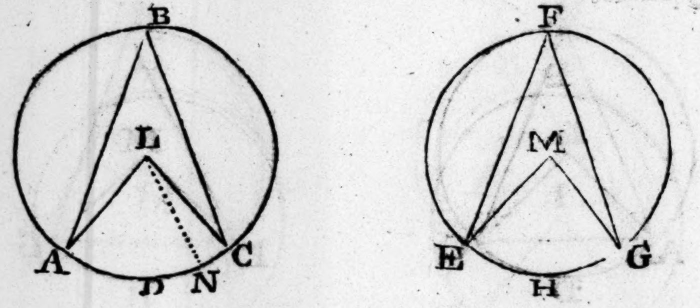
For find L and M the centers of the circles, and join AL, LC, EM, MG.

Now the angle ALC is double to ABC ——— 139
And the angle EMG is double to EFG ——— 139
Th. the angle ALC is equal to EMG ——— 51
And the ſides AL, LC are equal to EM, MG ——— 55
Th. the baſes AC, EG are equal ——— ——— 62
Th. the ſegments ABC, EFG are equal ——— 144
Th. the arches ADC, EHG are equal ——— 50

146. Corollary.

In equal circles, the ſectors of equal angles are equal to one another, namely ALCDA = EMGHE ——— ——— 53

27&3 147. In equal circles, angles ſtanding upon equal arches are equal, either at the centers or circumferences.



In the equal circles A B C D, E F G H, let the arches A D C, E H G be equal: then the angles A B C, E F G conſtituted thereon are alſo equal.

For find L and M the centers of the circles, join A L, L C, E M, M G, and if the angles A L C, E M G are not equal, make the angle A L N equal to E M G. ——— ——— ——— p. 9

Now the circle A B C D is equal to E F G H ——— theo.

And the angle A L N is equal to E M G ——— con.

Th. the arch A D N is equal to E H G ——— 145

But the arch A D C is equal to E H G ——— theo.

Th. the arch A D N is equal to A D C ——— 48

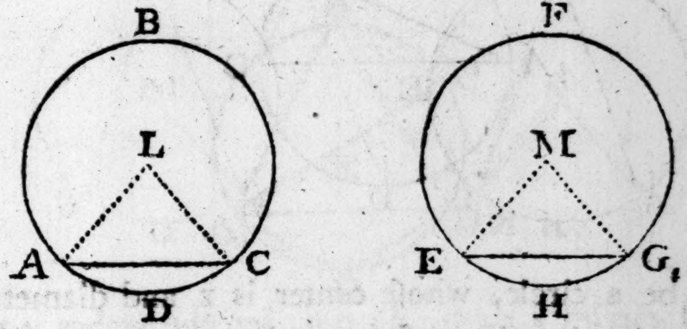
Which is impoſſible ——— ——— ——— 52

Th. the angles A L C, E M G muſt be equal ——— ———

And the angles A B C, E F G are the halves of A L C, E M G 139

Th. the angles A B C, E F G are equal ——— ——— 51

148. In equal circles, equal lines cut off equal parts of the circumferences, the leſſer arch equal to the leſſer, and the greater arch equal to the greater. 28l.3.



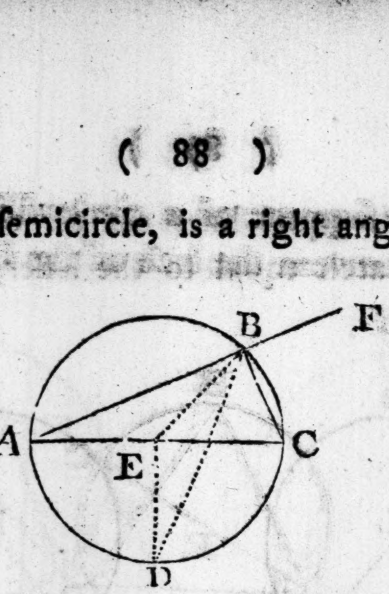
Let A B C D, E F G H be equal circles, and A C, E G be equal lines cutting off the leſſer parts A D C, E H G of their circumferences: then the arch A D C is equal to the arch E H G, and the arch A B C is equal to E F G.

For find L and M the centers of the circles, and join A L, L C, E M, M G.

Now the ſides A L, L C are equal to E M, M G ———	65
And the baſes A C, E G are equal ——— ———	theo.
Th. the angles A L C, E M G are equal ———	65
Th. the arch A D C is equal to the arch E H G ———	145
And the arch A B C is equal to the arch E F G ———	50

149. An

31 E 3. 149. An angle in a ſemicircle, is a right angle.



Let A B C D be a circle, whoſe center is E and diameter is A C, and to any point B in the circumference draw A B, C B: then A B C is a right angle.

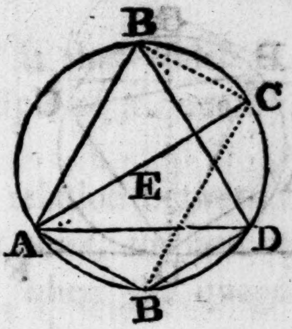
For join E B, and produce A B to F.

Now E A, E B, E C are equal —— —— 30
Th. the angle A B E is equal to E A B —— —— 62
And the angle E B C is equal to B C E —— 62
Th. the angle A B C is equal to E A B + B C E —— 49
But the angle C B F is equal to E A B + B C E —— 84
Th. the angle A B C is equal to C B F —— —— 48
Th. A B C is a right angle —— —— 9

Otherwiſe, draw E D at right angles to A C, and join B D.

Here, the angle A B D is equal to half A E D —— 139
And the angle D B C is equal to half D E C —— 139
But A E D, D E C are right angles —— —— con.
Th. A B C is a right angle —— —— —— 52

150. An angle in the ſegment of a circle differs from a right one, by the angle contained between the baſe of the ſegment and a diameter drawn from its extremity.



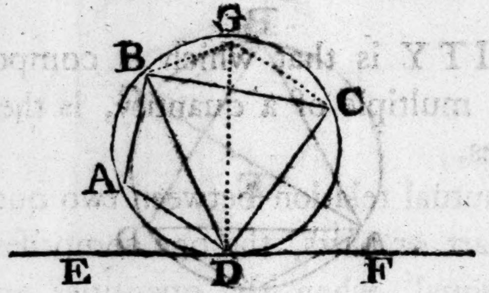
Let the circle ABCD be cut by the right line AD, and draw AB, BD and the diameter AC: then the angle ABD differs from a right angle by CAD.

For join BC.

Now ABC is a right angle. ——— ——— 149
And ABD differs from it by the angle DBC ——— ———
But the angle CAD is equal to the angle DBC ——— 140
Therefore, the angle ABD differs from a right one, by the angle CAD.

N 151. The

32ℓ J .151. The angles made by a tangent and a line drawn from the point of contact cutting the circle, are equal to the angles contained in the oppoſite ſegments of the circle.



Let the circle ABCD have a tangent EF touching it in D, draw DB at pleaſure cutting the circle, and join BC, CD, DA, AB; then the angles EDB, BDF are equal to the angles BCD, DAB each to each.

For draw DG at right angles to EF, and join BG, GC.

Now the line DG is a diameter of the circle ——— 138
Th. GCD is a right angle ——— ——— 149
Th. the angle EDG ——— = GCD ——— ——— 54
But the angle BDG ——— = GCB ——— ——— 140
Th. the angle EDB ——— = BCD ——— ——— 50
Ag. the angle EDB + BDF = 2 right angles ——— 66
And the angle BCD + DAB = 2 right angles ——— 141
Th. the angle EDB + BDF = BCD + DAB ——— 48
Th. the angle ——— BDF = ——— DAB ——— 50

Part

PART II.

152. QUANTITY is that which is compoſed of parts.

153. A multiple of a quantity, is the ſaid quantity taken any number of times.

154. Ratio is a mutual relation between two quantities of one kind, according to what part or parts, the one compriſes of the other..

155. Ratios are equal, when the quantities are the ſame part or parts of one another, in each pair.

Thus, the ratio between 3 and 6, is equal to the ratio between 4 and 8, and the ratio between 1 and 3, is equal to the ratio between a foot and a yard.

156. Proportion is an equality of ratios.

Thus the numbers 3, 6, 4, 8 are proportional; and if A, B repreſent quantities of one kind, and C, D other quantities of one kind, having an equal ratio with A and B, thoſe four quantities are proportional, and wrote A : B :: C : D, and read, as A to B, ſo is C to D.

Note. The firſt and third terms or quantities in a proportion, are called antecedents, and the ſecond and fourth conſequents.

157. A mean proportional, is a quantity that takes the ſecond and third places in a proportion. Thus, if A is to B as B to C, the quantity B is a mean proportional between A and C.

158. Alternate ratio, is when the antecedents are compared together, and the conſequents together.

159. Compounded ratio, is when the ſum of the antecedent and conſequent, is compared with either of them.

160. Di-

160. Divided ratio, is when the difference of the anteccedent and conſequent, is compared with either of them.

161. Similar right lined figures, are thoſe which have all their angles equal to one another, and the ſides about the equal angles proportional.

Axioms.

162. Equal quantities have equal ratios to the ſame quantity, or to equal quantities.

163. Quantities that have equal ratios to the ſame quantity, or to equal quantities, are equal to one another.

164. If two quantities be compared with any third quantity, the greater of them will have a greater ratio to it, than the leſſer.

165. If two quantities be compared with any third quantity, that which has the greater ratio to it, is the greater quantity.

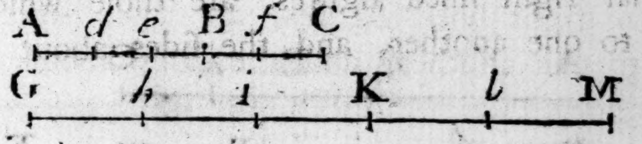
166. Any quantity may be divided into equal parts, that ſhall be leſs than any given quantity of the ſame kind.

167. Equimultiples of equal quantities are equal.

168. Any multiple of the whole, is equal to the ſame multiple of all its parts taken together.

169. Ratios that are equal to the ſame ratio are equal to one another, and may be taken the one for the other.

170. Quantities are proportional to their equimultiples.

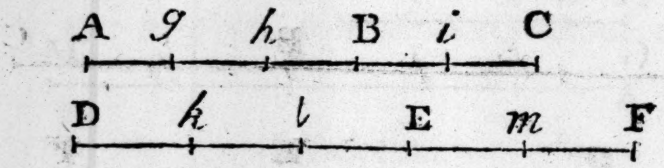


Let AB, BC be quantities of the ſame kind, and n any number whatſoever; then AB : BC :: AB × n : BC × n.

For conceive AB, BC divided into parts equal to one another at d, e, f, &c. and alſo that Gh, hi, iK, &c. are equal to n times Ad, de, eB, &c. in order.

Now Ad, de, eB, &c. are equal ——— ——— con.
Th. Gh, hi, iK, &c. are equal ——— ——— 167
And the number of parts in GK, KM are equal to the number in AB, BC each to each ——— ——— con.
Th. AB, BC and GK, KM have equal ratios ——— 155
Th. AB : BC :: GK : KM ——— ——— 156
But AB × n = GK, and BC × n = KM ——— 168
Th. AB : BC :: AB × n : BC × n ——— ——— 56

171. If four quantities of the ſame kind are proportional, they will be alternately proportional.

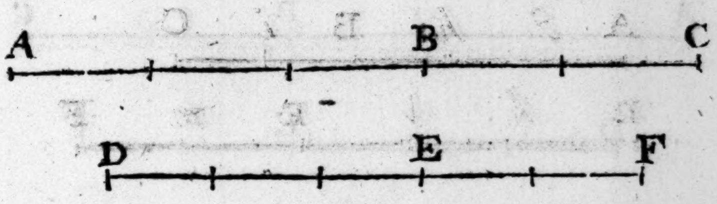


Let AB, BC, DE, EF be quantities of the ſame kind, and AB : BC :: DE : EF; then AB : DE :: BC : EF.

For conceive AB, BC divided into parts equal to one another, at g, h, i, &c. and alſo DE, EF divided into the ſame number of equal parts with AB, BC reſpectively, at k, l, m, &c.

Now AB : BC :: DE : EF ——— ——— theo.
Th. AB, BC and DE, EF have equal ratios ——— 156
Th. the parts in DE, EF are of equal magnitude ——— 155
Th. Ag : Dk :: AB : DE ——— ——— 170
And Ag : Dk :: BC : EF ——— ——— 170
Th. AB : DE :: BC : EF ——— ——— 169

172. If four quantities are proportional, they will alſo be proportional by compoſition, or addition.

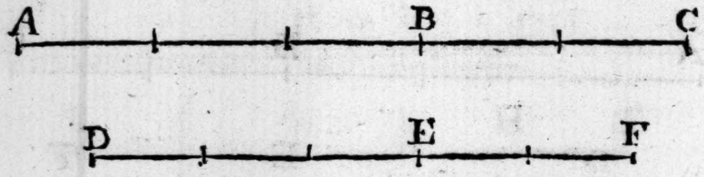


If the quantities AB, BC; DE, EF be proportional, and AB : BC :: DE : EF; then AB : AC :: DE : DF.

For conceive AB, BC divided into parts equal to one another, and DE, EF divided into the ſame number of equal parts with AB, BC reſpectively.

Now AB : BC :: DE : EF	theo.
Th. AB, BC and DE, EF have equal ratios	156
Th. the parts in DE, EF are of equal magnitude	155
Th. AB, AC and DE, DF have equal ratios	155
Th. AB : AC :: DE : DF	156

173. If four quantities are proportional, they will alſo be proportional by diviſion, or ſubſtraction.

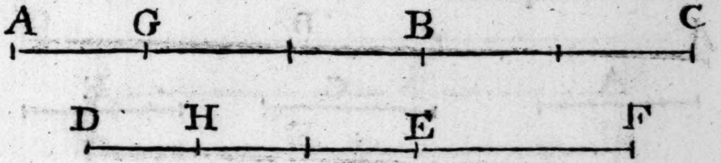


Let the quantities AC, AB, DF, DE be proportional, and AC : AB :: DF : DE; then AB : BC :: DE : EF.

For conceive AB, BC divided into parts equal to one another, and DE, EF divided into the ſame number of equal parts with AB, BC, each to each.

Now AC : AB :: DF : DE ——— ——— theo.
Th. AC, AB and DF, DE have equal ratios ——— 156
Th. the parts in DF, DE are of equal magnitude ——— 155
Th. AB, BC and DE, EF have equal ratios ——— 155
Th. AB : BC :: DE : EF ——— ——— 156

174. **If four quantities are proportional, their ſums and differences will be proportional.**

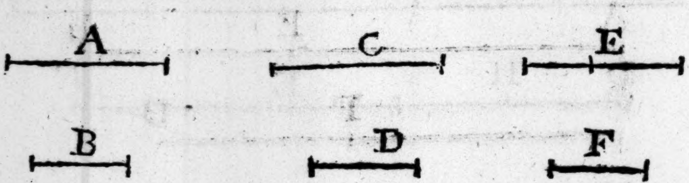


Let the quantities AB, BC, DE, EF be proportional AB : BC :: DE : EF, and let AB be greater than BC, take BG, EH equal to BC, EF reſpectively, then AC, AG and DF, DH are the ſums and differences of the propoſed quantities: I ſay, AC : AG :: DF : DH.

For conceive AG, GB, BC divided into parts equal to one another, and DH, HE, EF divided into the ſame number of equal parts with AG, GB, BC each to each.

Now AB : BC :: DE : EF —— —— theo.
Th. AB, BC and DE, EF have equal ratios —— 156
Th. the parts in DH, HE, EF muſt be of equal magnitude. 155
Th. AC. AG and DF, DH have equal ratios —— 155
Th. AC : AG :: DF : DH —— —— 156

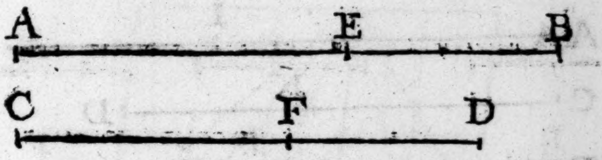
175. If any number of quantities of one kind are proportional, as one antecedent to its conſequent, ſo is the ſum of all the antecedents to the ſum of all the conſequents.



Let A, B, C, D, E, F be quantities of one kind, and A : B :: C : D :: E : F; then A : B :: A + C + E : B + D + F.

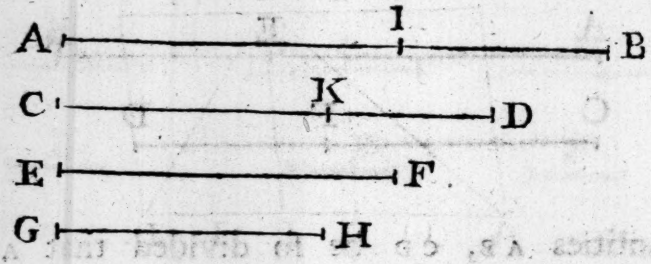
For A : C :: B : D ——— ——— ——— 171
Th. A : A + C :: B : B + D ——— ——— 172
Th. A : B :: A + C : B + D ——— ——— 171
But A : B :: E : F ——— ——— ——— theo.
Th. A + C : B + D :: E : F ——— ——— ——— 169
Th. A + C : E :: B + D : F ——— ——— 171
Th. A + C : A + C + E :: B + D : B + D + F 172
Th. A + C : B + D :: A + C + E : B + D + F ——— 171
Th. A : B :: A + C + E : B + D + F ——— ——— 169

176. If one quantity be to another, as a part of the one to a part of the other; thoſe quantities and their remaining parts are proportional, and alſo the parts themſelves.



Let the quantities AB, CD be ſo divided that AB : CD :: AE : CF; Then AB : CD :: EB : FD, and AE : CF :: EB : FD.
Now AB : CD :: AE : CF ——— ——— theo.
Th. AB : AE :: CD : CF ——— ——— 171
Th. AB : EB :: CD : FD —— —— —— 173
Th. AB : CD :: EB : FD —— —— —— 171
And AE : CF :: EB : FD —— —— —— 169

177. If four quantities of one kind be proportional, the greateſt and leaſt taken together, are greater than the other two taken together.

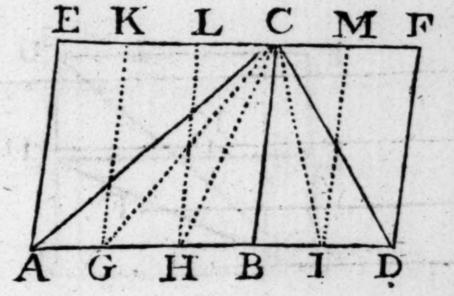


Let AB, CD, EF, GH be quantities of one kind, AB the greateſt, GH the leaſt, and AB : CD :: EF : GH; then AB + GH is greater than CD + EF.

For take AI equal to EF, and CK equal to GH.

Now AB : CD :: EF : GH — — — theo.
Th. AB : CD :: AI : CK — — 56
Th. AB : CD :: IB : KD — — 176
Th. AB, CD and IB, KD have equal ratios — 156
But AB is greater than CD — — theo.
Th. IB is greater than KD — — 155
Ag. AI + GH is equal to EF + CK — — 49
Th. AI + GH + IB is greater than EF + CK + KD — 49
Or AB + GH is greater than CD + EF — — 56

178. Triangles and parallelograms having the ſame altitude, are proportional to their baſes.



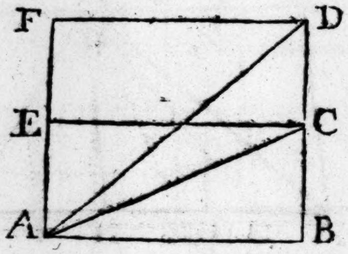
Let the triangles ABC, CBD and parallelograms EB, BF have the ſame altitude; then AB : BD :: ABC : CBD, and AB : BD :: EB : BF.

For conceive the baſes AB, BD divided into parts equal to one another at G, H, I, &c. From each point of diviſion draw a line to C the vertex of the triangles, and another parallel to AE or DF.

Now the parts AG, GH, HB, &c. are equal ——— con.
Th. the triangles ACG, GCH, HCB, &c. are equal —— 94
And the parallelograms AK, KH, HC, &c. are equal — 92
Th. AB, BD and the triangles ABC, CBD have equal ratios. 155
Alſo AB, BD and the parallelograms EB, BF have equal ratios. 155
Th. AB : BD :: ABC : CBD ——— ——— 156
And AB : BD :: EB : BF ——— ——— 156

179. Corollary.

Triangles and parallelograms having the ſame baſe, are proportional to their altitudes.



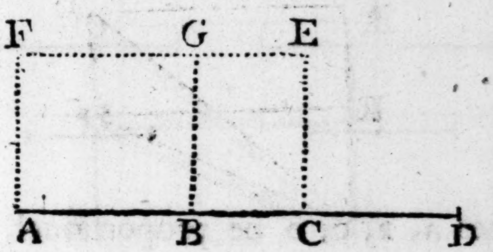
Since all parallelograms, and all triangles, conſtituted upon the ſame baſe and between the ſame parallels are equal by 90, 91, let the triangles ABC, ABD, and parallelograms BE, BF be right angled at B: then AB is their common baſe, and BC, BD are their reſpective altitudes ——— ——— ——— 29

Now if BC, BD be conſidered as baſes, the triangles and parallelograms have the ſame altitude AB ——— ——— ———

Th. BC : BD :: ABC : ABD ——— ——— 178

And BC : BD :: BE : BF ——— ——— 178

180. Of any three ſtrait lines, the firſt is to the ſecond, as a rectangle contained by the firſt and third to a rectangle contained by the ſecond and third.

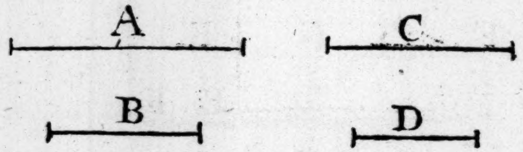


Let AB, BC, CD be lines taken at pleaſure; I ſay, AB : BC :: AB × CD : BC × CD.

For let ABC be a ſtrait line, erect CE perpendicular to AC and equal to CD, draw EF parallel to CA, and AF, BG parallel to CE.

Now AG, BE are rectangles —— —— 90
Th. AB : BC :: AG : BE —— —— 178
Or AB : BC :: AB × BG : BC × CE —— 61
But BG and CE are equal —— —— 89
Th. AB : BC :: AB × CE : BC × CE —— —— 56
Again, CE is equal to CD —— —— —— con.
Th. AB : BC :: AB × CD : BC × CD —— —— 56

181. If four ſtrait lines are proportional, the rectangle contained by the extremes, will be equal to the rectangle contained by the middle terms.



Let the ſtrait lines A, B, C, D be proportional, and $A : B :: C : D$. Then $A \times D = B \times C$.

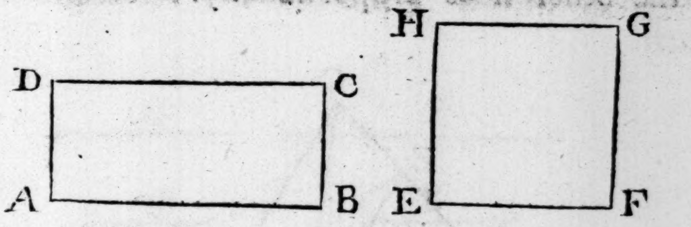
For $A : B :: A \times D : B \times D$ —— —— —— 180
Th. $C : D :: A \times D : B \times D$ —— —— —— 169
Ag. $C : D :: C \times B : D \times B$ —— —— —— 180
Th. $A \times D : B \times D : C \times B : D \times B$ —— —— 169
Th. $A \times D = B \times C$ —— —— —— 163

182. Corollary.

If three ſtrait lines are proportional, the rectangle contained by the extremes, will be equal to the ſquare of the middle term.

Thus if $A : B :: B : C$ —— —— —— theo.
Then $A \times C = B^2$ —— —— —— 181

183. The ſides of equal rectangles, are reciprocally proportional.

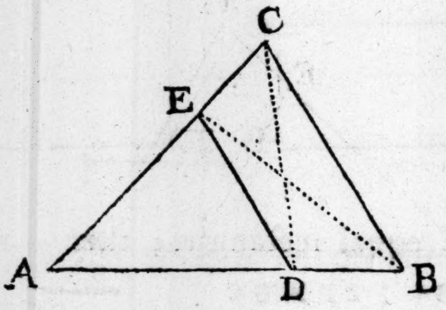


Let ABCD, EFGH be equal rectangles; then AB : EF :: FG : BC.
For AB : EF :: AB × BC : EF × BC —— —— 180
And AB × BC = EF × FG —— —— theo.
Th. AB : EF :: EF × FG : EF × BC —— 56
Ag. FG : BC :: EF × FG : EF × BC —— 180
Th. AB : EF :: FG : BC —— —— 169

184. Corollary.

If four lines are proportional, they will be alternately proportional.
Thus, if AB : EF :: FG : BC —— —— theo.
Then AB × BC = EF × FG —— —— 181
Th. AB : FG :: EF : BC —— —— 183

185. A line drawn within a triangle parallel to one of its ſides, will divide the other ſides proportionally.



In the triangle ABC, let the line DE be drawn parallel to the ſide BC; then AD : DB :: AE : EC, and AD : AE :: DB : EC.

For join BE, CD.

Now AD : DB :: ADE : EDB ——— ——— 178

And AE : EC :: ADE : EDC ——— ——— 178

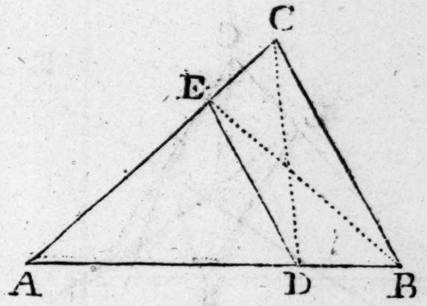
But the triangles EDB, EDC are equal ——— ——— 93

Th. AE : EC :: ADE : EDB ——— ——— 56

Th. AD : DB :: AE : EC ——— ——— 169

And AD : AE :: DB : EC ——— ——— 171

186. If two ſides of a triangle are divided proportionally, a line joining the points of diviſion will be parallel to the other ſide.



Let A B C be a triangle whoſe ſides A B, A C are divided proportionally at D and E, ſo that A D : D B :: A E : E C, and join D E; then D E is parallel to B C.

For join B E, C D.

Now A D : D B :: A E : E C ——— ——— theo.

And A D : D B :: A D E : E D B ——— ——— 178

Th. A E : E C :: A D E : E D B ——— ——— 169

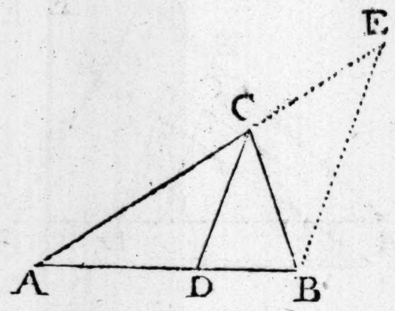
Ag. A E : E C :: A D E : E D C ——— ——— 178

Th. A D E : E D B :: A D E : E D C ——— ——— 169

Th. the triangles E D B, E D C are equal ——— 163

Th. D E is parallel to B C ——— ——— 95

187. If the vertical angle of a triangle is biſected by a line meeting the baſe, the parts of the baſe will be proportional to the ſides of the triangle.

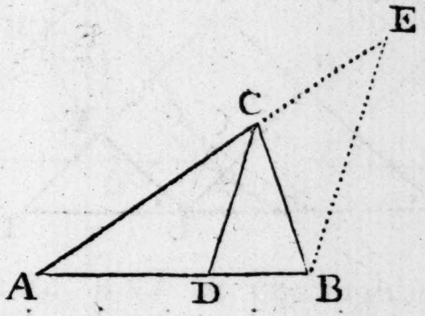


Let ABC be a triangle having its vertical angle ACB equally divided by the right line CD: then AC : CB :: AD : DB.

For produce AC to E, and draw BE parallel to DC.

Now the angle ACD is equal to CEB ——— 81
And the angle ACD is equal to DCB ——— theo.
Th. the angle CEB is equal to DCB ——— 48
Ag. the angle CBE is equal to DCB ——— 80
Th. the angles CEB, CBE are equal ——— ——— 48
Th. the ſides BC, CE are equal ——— ——— 64
But AC : CE :: AD : DB ——— ——— ——— 185
Th. AC : CB :: AD : DB ——— ——— ——— 56

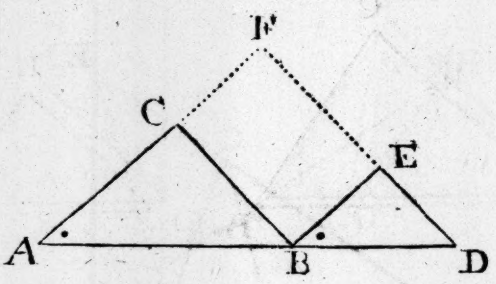
188. If the baſe of a triangle is divided in proportion to the ſides, a line joining the vertex and point of diviſion will biſect the vertical angle.



Let the triangle ABC have its baſe ſo divided in D, that AC : CB :: AD : DB, and join CD, then the angles ACD, DCB are equal.

For produce AC to E, and draw BE parallel to DC — p. 10
Now AC : CE :: AD : DB — — 185
And AC : CB :: AD : DB — — theo.
Th. AC : CE :: AC : CB — — 169
Th. CE is equal to CB — — 163
Th. the angle CBE is equal to CEB — — 63
But the angle ACD is equal to CEB — — 81
Th. the angle CBE is equal to ACD — — 48
Ag. the angle CBE is equal to DCB — — 80
Th. the angle ACD is equal to DCB — — 48

189. In equiangular triangles, the ſides about the equal angles are proportional, if taken correſpondently.



Let ABC, BDE be equiangular triangles, having the angle ABC equal to the angle BDE, and the angle BAC equal to DBE; then AB : AC :: BD : BE, AB : BC :: BD : DE, and AC : CB :: BE : ED.

For let ABD be a ſtrait line, and produce AC, DE, to F.

Now AF is parallel to BE, and DF is parallel to BC — 78

Th. BCFE is a parallelogram ——— ——— 22

Th. BC is equal to EF, and BE is equal to CF ——— 89

But AB : AC :: BD : CF ——— ——— 185

And AB : FE :: BD : ED ——— ——— 185

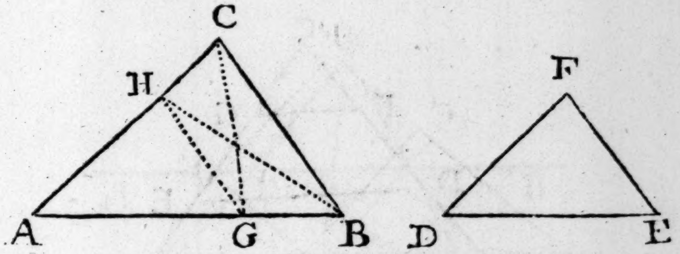
Th. AB : AC :: BD : BE ——— ——— 56

And AB : BC :: BD : DE ——— ——— 56

Alſo AC : CB :: BE : ED by the ſame reaſon ——— ———

189*. Other-

189*. Otherwiſe.

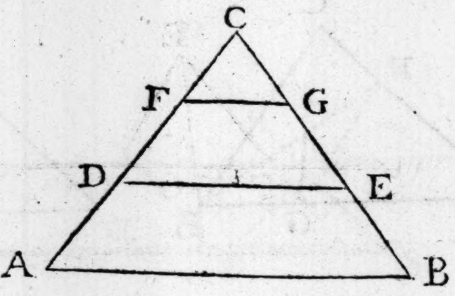


Let the triangles ABC, DEF be equiangular, having the angles ABC, CAB equal to the angles DEF, FDE each to each; then AB : DE :: AC : DF, AB : DE :: BC : EF, and AC : DF :: CB : FE.

For let AB be greater than DE, make AG, AH equal to DE, DF each to each, and join BH, HG, GC.

Now the angle AGH is equal to DEF ——— ——— 62
Th. the angle AGH is equal to ABC ——— 48
Th. GH is parallel to BC ——— ——— 78
Th. the triangles GHB, GHC are equal ——— 93
Th. the triangles ABH, AGC are equal ——— 49
But AB : AG :: ABH : AGH ——— ——— 178
Th. AB : AG :: AGC : AGH ——— ——— 56
Ag. AC : AH :: AGC : AGH ——— ——— 178
Th. AB : AG :: AC : AH ——— ——— 169
Th. AB : DE :: AC : DF ——— ——— 56
Alſo AB : DE :: BC : EF }
And AC : DF :: CB : FE } by the ſame reaſon.

190. If lines be drawn parallel to the baſe of a triangle cutting the ſides, the correſponding ſegments will be proportional.



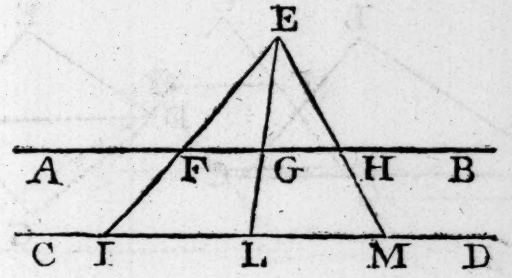
In the triangle ABC, let the lines DE, FG be drawn parallel to the baſe AB: then FD : DA :: GE : EB.

For the angles CAB, CDE, CFG are equal —— 81
And the angles CBA, CED, CGF are equal —— 81
Th. all the triangles are equiangular —— —— 27
Th. CF : CG :: CD : CE —— —— —— 189
And CF : CG :: FD : GE —— —— —— 185
Th. CD : CE :: FD : GE —— —— —— 169
Alſo CD : CE :: DA : EB —— —— —— 185
Th. FD : GE :: DA : EB —— —— —— 169
Th. FD : DA :: GE : EB —— —— —— 171

191. Corollary.

Hence again, if four right lines are proportional, they will alſo be proportional either compounded or divided.

192. If parallels be cut by many ſtrait lines proceeding from one point, their correſponding ſegments will be proportional.



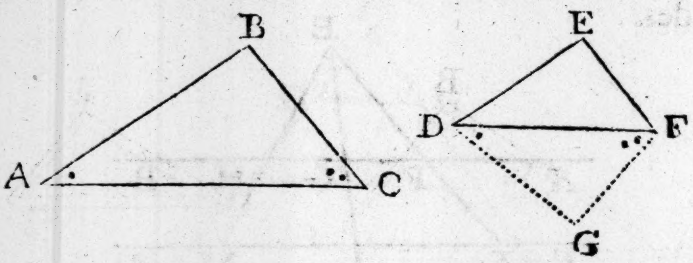
Let the parallels AB, CD be cut by ſtrait lines proceeding from the point E, in F, G, H, I, L, M: then FG : IL :: GH : LM.

For the angles EFG, EGF are equal to EIL, ELI —— 81
And the angle IEL is common —— —— ——
Th. the triangles EFG, EIL are equiangular —— 27
By the ſame reaſon, the triangles EGH, ELM are equiangular. —
Th. EG : EL :: FG : IL —— —— —— 189*
And EG : EL :: GH : LM —— —— 189*
Th. FG : IL :: GH : LM —— —— 169

193. Corollary. If one of the parallels is divided into equal parts, every other parallel will likewiſe be divided into equal parts.

For if FG = GH, then IL = LM —— —— 163

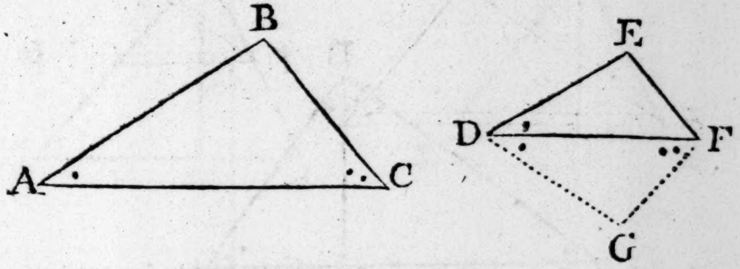
194. Triangles whoſe ſides are all proportional, are equiangular, having their correſponding angles equal to each other.



Let the triangles ABC, DEF have their ſides proportional, namely, AB : AC :: DE : DF, AB : BC :: DE : EF, and AC : CB :: DF : FE: then the angles BAC, ABC, ACB are equal to EDF, DEF, DFE each to each.

For make the angle FDG equal to the angle BAC, and the angle DFG equal to AÇB ——— ——— ——— p. 9
Hence, the angle DGF is equal to ABC ——— 85
Th. the triangles ABC, DGF are equiangular ——— 27
Th. AB : AC :: DG : DF ——— ——— 189
And AB : AC :: DE : DF ——— ——— theo.
Th. DG : DF :: DE : DF ——— ——— 169
Th. DG is equal to DE ——— ——— 163
By the ſame reaſon GF is equal to FE ——— ———
Th. the angle FDG = EDF, DGF = DEF, and DFG = DFE. 65
Th. the angle BAC = EDF, ABC = DEF, and ACB = DFE. 48

195. If triangles have an angle of the one equal to an angle of the other, and the ſides about them proportional; thoſe triangles are equiangular, having the angles equal which are oppoſite to correſponding ſides.



Let the triangles ABC, DEF have the angle BAC equal to the angle EDF, and BA : AC ∷ ED : DF : then the angles ABC, ACB are equal to DEF, DFE each to each.

For make the angle FDG equal to the angle BAC, and the angle DFG equal to ACB — — — P. 9

Then the angle DGF is equal to ABC — — 85

Th. BA : AC ∷ GD : DF — — — 189

But BA : AC ∷ ED : DF — — — theo.

Th. GD : DF ∷ ED : DF — — — 169

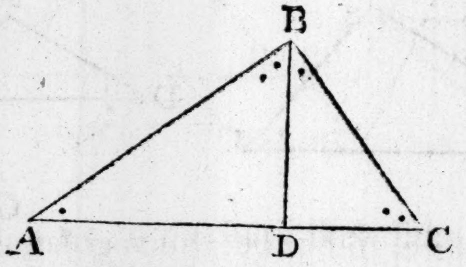
Th. GD is equal to DE — — — 163

And the angle GDF is equal to FDE — 48

Th. the angle DEF = DGF, and DFE = DFG — 62

Th. the angle ABC = DEF, and ACB = DFE — 48

196. In right angled triangles, a perpendicular falling from the right angle upon the baſe, will be a mean proportional between the ſegments made thereby, and each ſide will be a mean proportional between the baſe and adjacent ſegment.



If the triangle ABC has a right angle at B, and the line BD be drawn perpendicular to the baſe AC; then AD : DB :: BD : DC, AC : AB :: AB : AD, and AC : CB :: BC : CD.

For the angle ABC is equal to ADB ——— 54

And the angle BAC is common ——— ——— ———

Th. the angle ACB is equal to ABD ——— 85

By the ſame reaſon the angle BAC is equal to DBC. ———

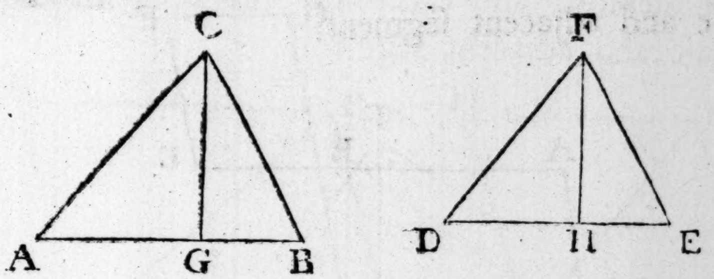
Th. the three triangles are equiangular ——— 27

Th. AD : DB :: BD : DC ——— ——— ——— 189

---- AC : AB :: AB : AD ——— ——— ——— 189

And AC : CB :: BC : CD ——— ——— ——— 189

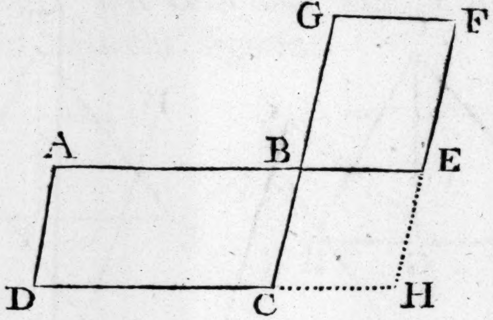
197. In equiangular triangles, the correſponding ſides and perpendiculars are proportionel.



Let the triangles ABC, DEF be equiangular, having the angle A equal to the angle D, the angle B equal to the angle E, and CG, FH perpendicular to the baſes AB, DE; then CG : AB :: FH : DE.

For the angle AGC is equal to DHF —— —— 54
And the angle A is equal to the angle D —— theo.
Th. the angle ACG is equal to DFH —— —— 85
And the angle ACB is equal to DFE —— —— 85
Th. AC : DF :: CG : FH —— —— —— 189
And AC : DF :: AB : DE —— —— —— 189
Th. CG : FH :: AB : DE —— —— —— 169
Th. CG : AB :: FH : DE —— —— —— 171

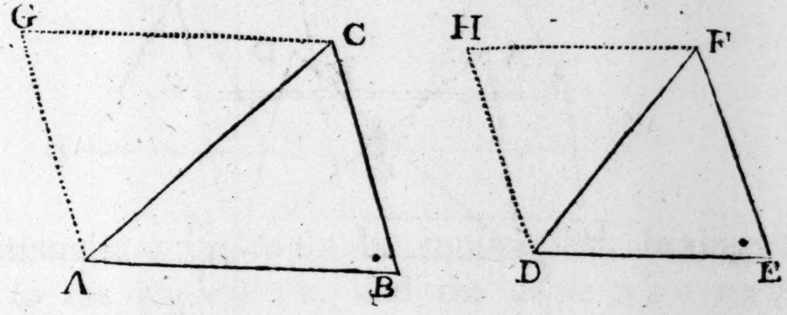
198. Equiangular parallelograms, are proportional to the rectangles contained by their ſides.



Let ABCD, BEFG be equiangular parallelograms, having the angle ABC equal to the angle GBE; then AB × BC : BE × EF :: AC : GE.

For let AB, BE be placed in a ſtrait line, and produce DC, FE to H.
Now the angle ABG + GBE = two right angles ——— 66
Th. the angle ABG + ABC = two right angles ——— 56
Th. GBC is a ſtrait line ——— ——— ——— 68
Th. BEHC is a parallelogram ——— ——— ——— 22
Th. AB : BE :: AC : BH ——— ——— ——— 178
But AB : BE :: AB × BC : BE × BC ——— ——— 180
Th. AB × BC : BE × BC : AC : BH ——— ——— 169
Th. AB × BC : AC :: BE × BC : BH ——— ——— 171
Alſo BE × EF : GE :: BE × BC : BH, by the ſame reaſon. ———
Th. AB × BC : AC :: BE × EF : GE ——— ——— 169
Th. AB × BC : BE × EF :: AC : GE ——— ——— 171

199. Triangles that have an angle of the one equal to an angle of the other, are proportional to rectangles within the ſides about thoſe equal angles.



Let the triangles ABC, DEF have the angle ABC equal to the angle DEF; then AB×BC : DE×EF :: ABC : DEF.

For compleat the parallelograms BG, EH ——— p. 10

Now ABC : DEF :: twice ABC : twice DEF ——— 170

And BG, EH are twice ABC, DEF ——— ——— 89

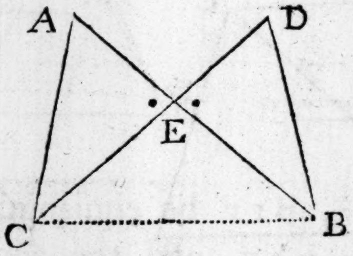
Th. ABC : DEF :: BG : EH ——— ——— ——— 56

But AB×BC : DE×EF :: BG : EH ——— ——— 198

Th. AB×BC : DE×EF :: ABC : DEF ——— 169

199*. Otherwiſe,

Draw the lines AB, CD at pleaſure interſecting at E, and join AC, BD; I ſay as before, AE × EC : BE × ED ∷ triangle AEC : triangle BED; For draw CB.

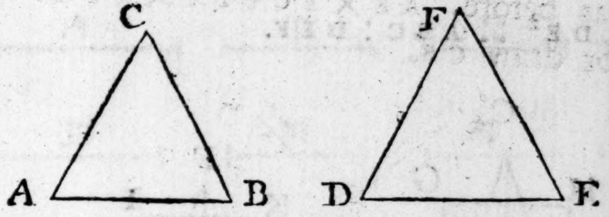


Now AE : EB ∷ AE × EC : EB × EC —— —— 180
And AE : EB ∷ AEC : CEB —— —— 178
Th. AE × EC : EB × EC ∷ AEC : CEB —— 169
Th. AE × EC : AEC ∷ EB × EC : CEB —— 171
Alſo BE × ED : BED ∷ EB × EC : CEB, by the ſame reaſon. —
Th. AE × EC : AEC ∷ BE × ED : BED —— 169
Th. AE × EC : BE × ED ∷ AEC : BED —— 171

200. Hence, If the rectangles within the ſides about the equal angles are equal, the triangles will be equal, and thoſe ſides reciprocally proportional.

For let AE × EC be equal to BE × ED —— theo.
Then the triangle AEC is equal to BED —— 163
And AE : EB ∷ DE : EC —— —— 183

201. Equiangular triangles, are proportional to the ſquares of their correſponding ſides.

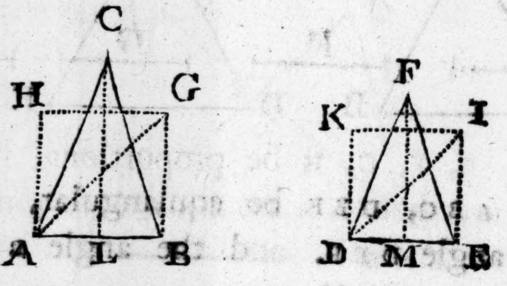


Let the triangles ABC, DEF be equiangular, having the angle ABC equal to the angle DEF, and the angle BAC equal to the angle EDF; then $AB^2 : DE^2 :: ABC : DEF$.

For $AB : BC :: AB^2 : AB \times BC$ 180
And $DE : EF :: DE^2 : DE \times EF$ 180
But $AB : BC :: DE : EF$ 189
Th. $AB^2 : AB \times BC :: DE^2 : DE \times EF$ 169
Th. $AB^2 : DE^2 :: AB \times BC : DE \times EF$ 171
Alſo $ABC : DEF :: AB \times BC : DE \times EF$ 199
Th. $AB^2 : DE^2 :: ABC : DEF$ 169

201*. Otherwiſe.

Let ABC, DEF be equiangular triangles, having the angle ABC equal to the angle DEF, and the angle BAC equal to the angle EDF; then AB^2 : DE^2 :: ABC : DEF.



For upon the baſes AB, DE let fall the perpendiculars CL, FM by problem 7, deſcribe the ſquares ABGH, DEIK by problem 11, and join AG, DI.

Now BG : CL :: ABG : ABC — — 179

And AB is equal to BG — — 24

Th. AB : CL :: ABG : ABC — — 56

And DE : FM :: DEI : DEF, by the ſame reaſon — —

But AB : CL :: DE : FM — — 197

Th. ABG : ABC :: DEI : DEF — — 169

Th. ABG : DEI :: ABC : DEF — — 171

Alſo ABG : DEI :: ABGH : DEIK — — 170

Th. ABC : DEF :: ABGH : DEIK — — 169

Or AB^2 : DE^2 :: ABC : DEF — — —

202. The rectangles contained by the correſponding terms in two ranks of proportional lines, are proportional.

Let A, B, C, D; E, F, G, H be proportional lines, and A : B :: C : D; E : F :: G : H; then A × E : B × F :: C × G : D × H.

For A : B :: A × E : B × E ——— ——— 180

And C : D :: C × G : D × G ——— ——— 180

Th. A × E : B × E :: C × G : D × G ——— ——— 169

Th. A × E : C × G :: B × E : D × G ——— ——— 171

Ag. E : F :: B × E : B × F ——— ——— 180

And G : H :: D × G : D × H ——— ——— 180

Th. B × E : B × F :: D × G : D × H ——— ——— 169

Th. B × E : D × G :: B × F : D × H ——— ——— 171

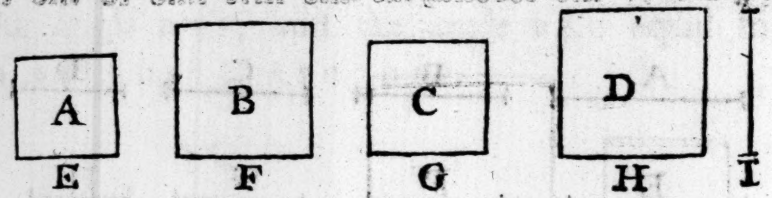
Th. A × E : C × G :: B × F : D × H ——— ——— 169

Th. A × E : B × F :: C × G : D × H ——— ——— 171

203. Corollary.

The ſquares of proportional lines, are proportional.

204. If ſquares be proportional, their ſides are alſo proportional.

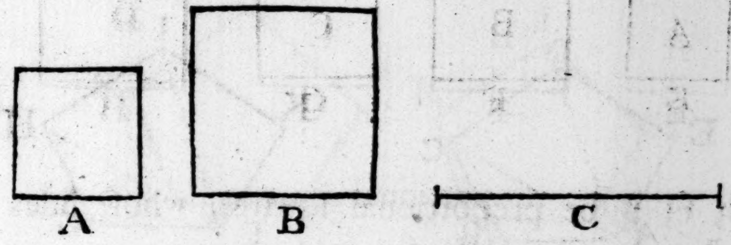


Let A, B, C, D be proportional ſquares, whoſe ſides are E, F, G, H, and A : B :: C : D; then E : F :: G : H.

For take the line I, a fourth proportional to E, F and G, by problem 5.

Now E : F :: G : I ——— ——— ——— con.
Th. A : B :: C : I^2 ——— ——— ——— 203
But A : B :: C : D ——— ——— ——— theo.
Th. C : I^2 :: C : D ——— ——— ——— 169
Th. I^2 is equal to D ——— ——— ——— 163
Th. the lines H and I are equal ——— ——— 103
Th. E : F :: G : H ——— ——— ——— 56

205. If three ſtrait lines are proportional, the ſquare of the firſt is to the ſquare of the ſecond, as the firſt line to the third.



Let A, B, C be proportional lines, and $A : B :: B : C$: Then $A^2 : B^2 :: A : C$.

For $A : B :: A^2 : B \times A$ ——— ——— 180

And $B : C :: B^2 : C \times B$ ——— ——— 180

But $A : B :: B : C$ ——— ——— ——— theo.

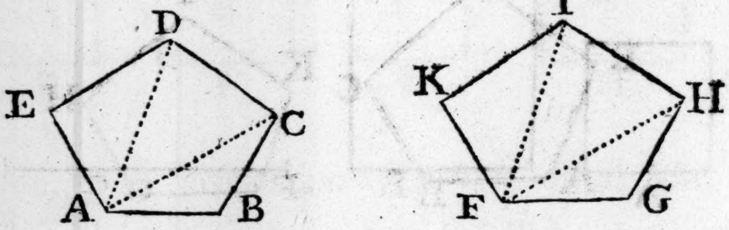
Th. $A^2 : B \times A :: B^2 : C \times B$ ——— ——— 169

Th. $A^2 : B^2 :: B \times A : C \times B$ ——— ——— 171

Ag. $A : C :: B \times A : C \times B$ ——— ——— 180

Th. $A^2 : B^2 :: A : C$ ——— ——— ——— 169

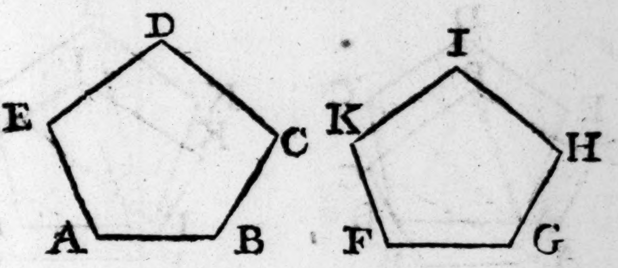
206. Similar polygons are diviſible into equal numbers of triangles, each triangle in the one equiangular to its correſpondent in the other.



Let the polygons ABCDE, FGHIK be ſimilar, having the angle ABC equal to the angle FGH, the angle BCD equal to the angle GHI, &c. and join AC, AD, FH, FI; then the triangle ABC is equiangular to the triangle FGH, the triangle ACD is equiangular to FHI, &c.

For the angle ABC is equal to FGH —— —— 161
And AB : BC :: FG : GH —— —— —— 161
Th. the triangles ABC, FGH are equiangular —— 195
And the angles ACB, FHG are equal —— 195
Th. the angles ACD, FHI are equal —— 50
Again, BC : GH :: AC : FH —— —— 189
And BC : GH :: CD : HI —— —— 161
Th. AC : FH :: CD : HI —— —— 169
Th. the triangles ACD, FHI are equiangular —— 195
By the ſame reaſon ADE, FIK are equiangular —— ——

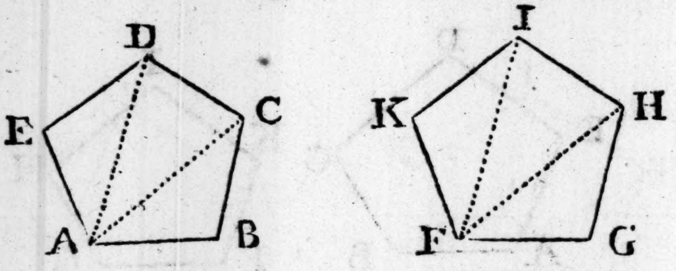
207. The circuits of ſimilar polygons, are proportional to their correſponding ſides.



Let the polygons ABCDE, FGHIK be ſimilar, having the angle ABC equal to the angle FGH, the angle BCD equal to the angle GHI, &c; then AB : FG :: the circuit of ABCD : circuit of FGHI.

For AB : FG :: BC : GH :: CD : HI, &c. —— 161
Th. AB : FG :: AB + BC + CD, &c : FG + GH + HI, &c. 175
But AB + BC + CD, &c. is the circuit of ABCDE — 52
And FG + GH + HI, &c. is the circuit of FGHIK — 52
Th. AB : FG :: circuit ABCDE : circuit FGHIK — 56

208. **The areas of ſimilar polygons, are proportional to the ſquares of their correſponding ſides.**



Let the polygons ABCDE, FGHIK be ſimilar, and have the angle ABC equal to the angle FGH, the angle BCD equal to GHI, &c. then $AB^2 : FG^2 :: ABCDE : FGHIK$.

For join AC, AD, FH, FI.

Now the triangles in one polygon are equiangular to thoſe in the other, each to its correſpondent —— —— 206

Th. $AB : FG :: AC : FH :: AD : FI$ —— —— 189

Th. $AB^2 : FG^2 :: AC^2 : FH^2 :: AD^2 : FI^2$ —— —— 203

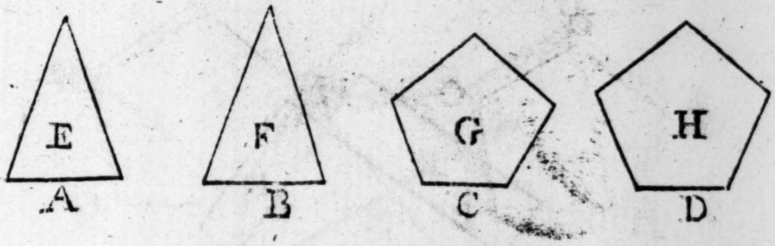
But $AB^2 : FG^2 :: ABC : FGH$, &c. —— —— 201

Th. $ABC : FGH :: ACD : FHI :: ADE : FIK$ —— 169

Th. $ABC : FGH :: ABCDE : FGHIK$ —— —— 175

Th. $AB^2 : FG^2 :: ABCDE : FGHIK$ —— —— 169

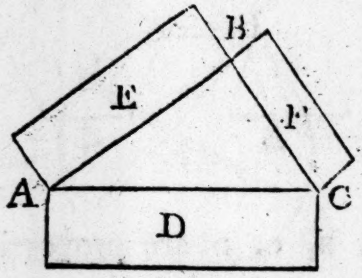
209. If four right lines are proportional, figures deſcribed upon them ſimilar and alike ſituated, will alſo be proportional.



If the right lines A, B, C, D are proportional $A : B :: C : D$, and upon A, B the figures E, F be deſcribed ſimilar to each other alike ſituated; and alſo upon C, D the figures G, H be deſcribed ſimilar to each other and alike ſituated: I ſay, $E : F :: G : H$.

For $A^2 : B^2 :: C^2 : D^2$ ——— ——— 203
And $A^2 : B^2 :: E : F$ ——— ——— ——— 208
Th. $C^2 : D^2 :: E : F$ ——— ——— ——— 169
Ag. $C^2 : D^2 :: G : H$ ——— ——— ——— 208
Th. $E : F :: G : H$ ——— ——— ——— 169

210. In right angled triangles, any figure deſcribed upon the ſide oppoſite to the right angle, will be equal to both the figures deſcribed upon the ſides containing it, if all are ſimilar and alike ſituated.



If the triangle ABC is right angled at B, and the figures D, E F deſcribed on its ſides be ſimilar to one another and alike ſituated; then D is equal to both E and F.

For $AB^2 : AC^2 :: E : D$ —— —— 208

Th. $AB^2 : E :: AC^2 : D$ —— —— 171

Ag. $AB^2 : BC^2 :: E : F$ —— —— 208

Th. $AB^2 : AB^2 + BC^2 :: E : E + F$ —— —— 172

Th. $AB^2 : E :: AB^2 + BC^2 : E + F$ —— —— 171

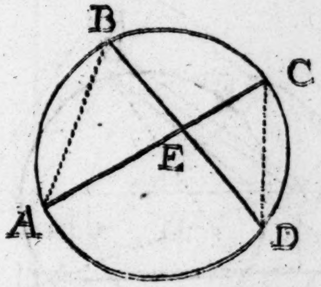
Th. $AC^2 : D :: AB^2 + BC^2 : E + F$ —— —— 169

But $AC^2 = AB^2 + BC^2$ —— —— —— 98

Th. $AC^2 : D :: AC^2 : E + F$ —— —— 56

Th. $D = E + F$ —— —— —— 163

211. If two lines within a circle cut each other and terminate in the circumference, the rectangle contained by the parts of the one, will be equal to the rectangle contained by the parts of the other.

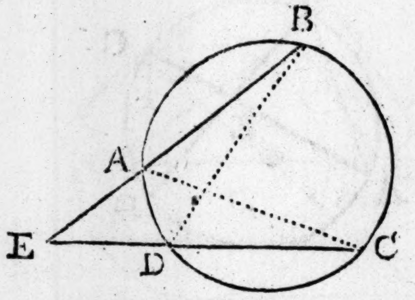


In the circle ABCD, let the lines AC, BD cut each other in E; then $AE \times EC = BE \times ED$.

For join AB, CD.

Now the angle AEB = CED —— —— 69
----- the angle ABD = ACD —— —— 140
And the angle BAC = BDC —— —— 140
Th. the triangles ABE, ECD are equiangular —— 27
Th. AE : EB :: DE : EC —— —— 189
Th. $AE \times EC = BE \times ED$ —— —— 181

212. If from any point without a circle, lines be drawn cutting it and terminating in the circumference; the rectangles contained by the wholes lines and their external parts will be equal to one another.

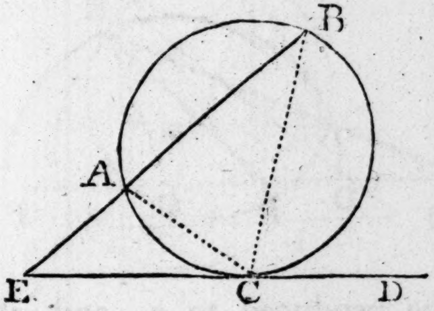


From the point E without the circle ABCD, let the lines EB, EC be drawn cutting the circumference in A and D; then EB × AE = EC × DE.

For join AC, BD.

Now the angle ABD is equal to ACD —— 140
And the angle BEC is common —— —— ——
Th. the angles CAE, EDB are equal —— —— 85
Th. the triangles EAC, EDB are equiangular —— 27
Th. AE : EC :: DE : EB —— —— —— 189
Th. EB × AE = EC × DE —— —— 181

213. If from any point without a circle, one line be drawn cutting the circle, and another touching it; the rectangle contained by the whole cutting line and its external part, will be equal to the ſquare of the tangent.

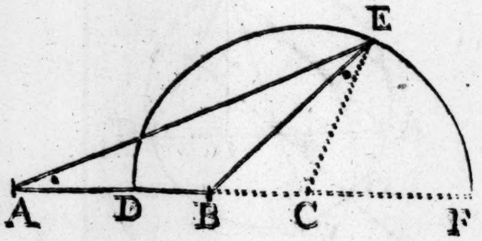


From the point E without the circle ABC, let the line EB be drawn cutting the circumference in A, and ED touching it in C; Then $EB \times AE = EC^2$.

For join AC, CB.

Now the angle ACE is equal to EBC ——— 151
And the angle AEC is common ——— ——— ———
Th. the angles CAE, ECB are equal ——— ——— 85
Th. the triangles ACE, EBC are equiangular ——— 27
Th. $AE : EC :: CE : EB$ ——— ——— ——— 189
Th. $EB \times AE = EC^2$ ——— ——— ——— 182

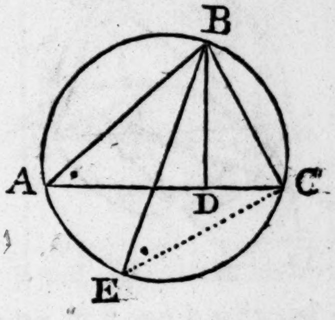
214. If a right line is cut by a circle into two ſuch parts, that their difference is to the leſſer, as the greater to the radius being the leſſer part produced: theſe parts will be proportional to lines joining any point in the circumference and the extremes of the ſaid line.



Let the line AB be produced to C, and about the center C let the circle DEF be deſcribed cutting AB in D, ſo that AD—DB : DB :: AD : DC, and join AE, EB: then AE : EB :: AD : DB. For join CE.

Now AD—DB : DB :: AD : DC ——— ——— theo.
Th. ——— AD : DB :: AC : DC ——— ——— 172
Th. ——— AD : AC :: DB : DC ——— ——— 171
Th. ——— AD : DC :: DB : BC ——— ——— 173
Th. ——— AC : DC :: DC : BC ——— ——— 172
But CE is equal to DC ——— ——— ——— 30
Th. ——— AC : CE :: EC : CB ——— ——— 56
And the contained angle ACE is common ——— ———
Th. the triangles ACE, ECB are equiangular ——— 195
Th. ——— AE : BE :: AC : CE ——— ——— 189*
Th. ——— AE : BE :: AC : DC ——— ——— 56
Th. ——— AE : BE :: AD : DB ——— ——— 169

215. The rectangle contained by the ſides of a triangle, is equal to the rectangle contained by a perpendicular to the baſe and diameter of a circumſcribing circle.

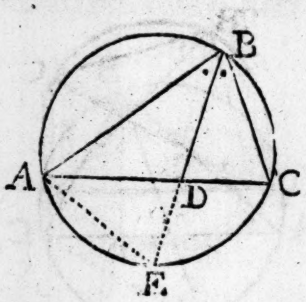


If a circle is deſcribed about the triangle ABC, and the perpendicular BD, and diameter BE be drawn; then $AB \times BC = EB \times BD$.

For join EC.

Now BCE is a right angle ——— ——— ——— 149
Th. the angles ADB, BCE are equal ——— ——— 54
And the angles BAD, BEC are equal ——— ——— 140
Th. the angles ABD, EBC are equal ——— ——— 85
Th. the triangles ABD, EBC are equiangular. ——— 27
Th. AB : BD :: EB : BC ——— ——— ——— 189
Th. $AB \times BC = EB \times BD$ ——— ——— ——— 181

216. The rectangle contained by the ſides of a triangle, is equal to the ſquare of a line biſecting the vertical angle and meeting the baſe, with a rectangle contained by the parts of the baſe.

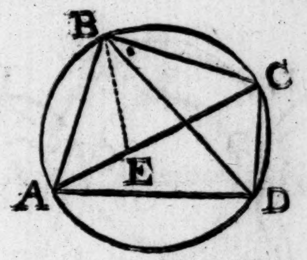


In the triangle ABC, let the line BD be drawn making the angles ABD, DBC equal, and meeting the baſe in D; then $AB \times BC = BD^2 + AD \times DC$.

For deſcribe a circle ABCE about the triangle by problem 34, produce BD to E, and join AE.

Now the angle $ABD = DBC$ —— —— theo.
And the angle $AEB = BCD$ —— —— 140
Th. the angle $BAE = BDC$ —— —— 85
Th. the triangles ABE, BCD are equiangular —— 27
Th. $AB : BE :: DB : BC$ —— —— 189
Th. $AB \times BC$ —— $= BE \times BD$ —— —— 181
But $BD^2 + DE \times BD = BE \times BD$ —— —— 106
Th. $AB \times BC$ —— $= BD^2 + DE \times BD$ —— 48
Ag. $AD \times DC$ —— $=$ —— $DE \times BD$ —— 211
Th. $AB \times BC$ —— $= BD^2 + AD \times DC$ —— 56

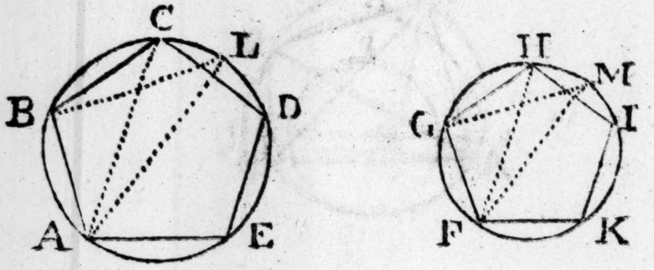
217. **If a quadrangle be inſcribed in a circle, the rectangle contained by its diagonals, will be equal to both the rectangles contained by the oppoſite ſides.**



Let the quadrangle ABCD be inſcribed in a circle, and draw the diagonals AC, BD; then $AC \times BD = AB \times CD + AD \times BC$.

For make the angle ABE equal to CBD —— p. 9
Now the angle ABD is equal to EBC —— 49
—— the angle BAC is equal to CDB —— 140
And the angle ADB is equal to BCA —— 140
Th. the triangles ABE, BCD are equiangular, and the triangles ABD, EBC are equiangular —— —— 85
Th. $BA : AE :: BD : DC$ —— —— 189
And $AD : DB :: EC : CB$ —— —— 189
Th. $AB \times CD = AE \times BD$ —— —— 181
And $AD \times BC = EC \times BD$ —— —— 181
Th. $AB \times CD + AD \times BC = AE \times BD + EC \times BD$ —— 49
But $AC \times BD$ —— $= AE \times BD + EC \times BD$ —— 104
Th. $AC \times BD$ —— $= AB \times CD + AD \times BC$ —— 48

218. The circuits of ſimilar polygons inſcribed in circles are proportional to the diameters, and their areas to the ſquares of the diameters.



Let ABCDE, FGHIK be ſimilar polygons inſcribed in circles, whoſe diameters are AL, FM.

Then AL : FM :: circuit ABCDE : circuit FGHIK.

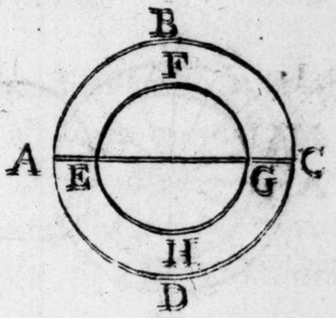
And $AL^2 : FM^2$ :: area of ABCDE : area of FGHIK.

For join AC, BL, FH, GM.

Now the angle BCA = BLA, and GHF = GMF —— 140
Alſo the angle BCA = GHF —— —— —— 206
Th. the angle BLA = (GHF =) GMF —— 48
And ABL, FGM are right angles —— 149
Th. the triangles ABL, FGM are equiangular —— 85
Th. AB : FG :: AL : FM —— —— —— 189
Th. $AB^2 : FG^2 :: AL^2 : FM^2$ —— —— 203
But AB : FG :: circuit ABCDE : circuit FGHIK —— 207
And $AB^2 : FG^2$ :: area of ABCDE : area of FGHIK — 208
Th. AL : FM :: circuit ABCDE : circuit FGHIK —— 169
And $AL^2 : FM^2$ :: area of ABCDE : area of FGHIK — 169

219. Corollary.

The circumferences of circles are proportional to their diameters, and the areas to the ſquares of their diameters.

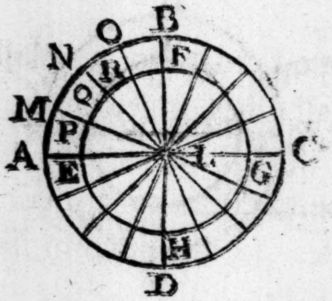


Thus let ABCD, EFGH be circles, whoſe diameters are AC, EG.
Then AC : EG :: circumf. ABCD : circumf. EFGH ——— 218
And $AC^2 : EG^2$ :: area of ABCD : area of EFGH ——— 218
Becauſe circles may be conſidered as agreeing with ſimilar inſcribed polygons, whoſe ſides are indefinitely ſmall.

220. Hence, if there be three ſuch circles, that the ſquare of one of their diameters, is equal to the ſum of the ſquares of the other two diameters, the greater circle is equal to both the leſſer. 210

219*. Otherwiſe,

The circumferences of circles are as the ſemidiameters, and the areas are as the ſquares of the ſemidiameters.



Let the circles ABCD, EFGH have a common center at L, and join AL; then AL : EL :: circumf. ABCD : circumf. EFGH, and ———-- $AL^2 : EL^2$ :: area of ABCD : area of EFGH.

For conceive the circumference ABCD divided into equal parts AM, MN, NO, &c. and draw LM, LN, LO, &c. cutting the leſſer circle in E, P, Q, &c.

Now the angles ALM, MLN, NLO, &c. are equal —— 147

Th. the arches EP, PQ, QR, &c. are equal —— 145

Alſo the ſectors ALM, MLN, NLO, &c. are equal —— 146

And the ſectors ELP, PLQ, QLR, &c. are equal —— 146

Th. AM : EP :: circumf. ABCD : circumf. EFGH — 170

And ALM : ELP :: area of ABCD : area of EFGH — 170

Again, let the parts AM, MN, NO, &c. diminiſh, till the ſectors ALM, ELP may be conſidered as equiangular triangles — 166

Then AM : EP :: AL : EL —— 189

And ALM : ELP :: $AL^2 : EL^2$ —— 201

Th. AL : EL :: circumf. ABCD : circumf. EFGH —— 169

And $AL^2 : EL^2$ :: area of ABCD : area of EFGH —— 169

PART III. OF *SOLIDS*.

221. A Line is perpendicular to a plain, when it makes right angles with all lines meeting it, which are drawn in the ſaid plain.

222. A plain is perpendicular to a plain, when all lines in the one drawn at right angles to their common ſection, are perpendicular to the other.

223. The inclination of a line to a plain, is the angle contained by it, and a line joining the points of ſection, made by the ſaid line, and a perpendicular falling on the plain from its extremity.

224. The inclination of a plain to a plain, is an acute angle contained by lines drawn in each plain, from any point in their common ſection at right angles to it.

225. Parallel plains, are thoſe which if infinitely produced, will never meet.

226. A pyramid, is contained by ſeveral plains ſtanding upon a plain, and meeting in one point.

227. A priſm, is contained by ſeveral plains, the two baſes of which are equal, ſimilar and parallel; but the others are parallelograms.

228. A cube, is contained by ſix equal ſquares.

229. A ſphere or globe, is generated by the revolution of a ſemicircle about its diameter, called the axis.

230. The center of a globe, is the ſame as the center of the generating ſemicircle.

231. The diameter of a globe, is a line paſſing thro' the center, and terminating in the ſurface.

232. A cone, is generated by the revolution of a ſtrait line about the circumference of a circle, and always paſſing thro' a fixed point above the plain of the ſaid circle.

233. The axis of a cone, is the line joining the fixed point and center of the circle.

234. A cylinder, is generated by the revolution of a ſtrait line, about the circumferences of two parallel and equal circles, and always parallel to a line joining their centers.

235. The axis of a cylinder, is the line joining the centers of the parallel circles.

236. Right cones and cylinders, are thoſe whoſe axes are perpendicular to their baſes.

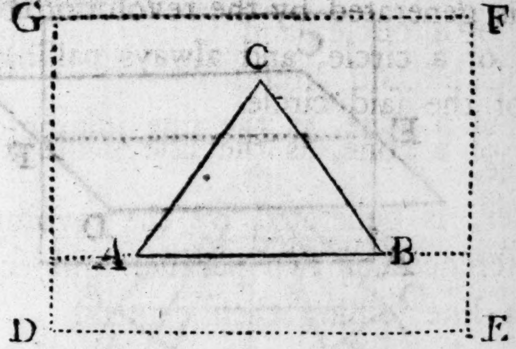
237. Right cones, and cylinders are ſimilar, when their axes are proportional to the diameters of their baſes.

238. Solids are ſimilar, which are contained by equal numbers of ſimilar plains, alike ſituated.

239. An Axiom.

If ſolids of equal baſe and altitude ſtanding upon the ſame plain, are cut by plains parallel to their baſes, and all the correſponding ſections poſſible be equal to each other, thoſe ſolids fill equal ſpaces, and are equal to one another.

240. The three ſides of a triangle, are in the ſame plain.



Let A B C be a plain triangle, then its ſides A B, B C, C A are in the ſame plain.

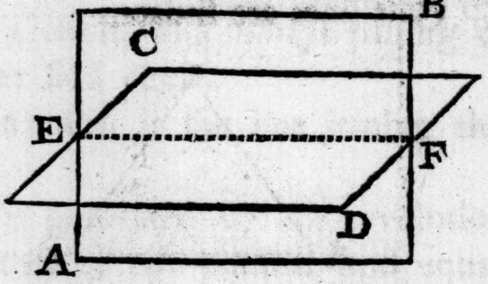
For conceive a plain D E F G paſſing through the points A and B, to revolve, or turn about the line A B as an axis, till it falls upon the point C, and there remains at reſt.

Now the points A, B, C are in the plain D E F G —— above

Therefore, the lines A B, B C, C A are alſo in the plain D E F G. 7

Conſequently, the three ſides of any plain triangle, are in one plain.

241. The common ſection of two plains, is a ſtrait line.



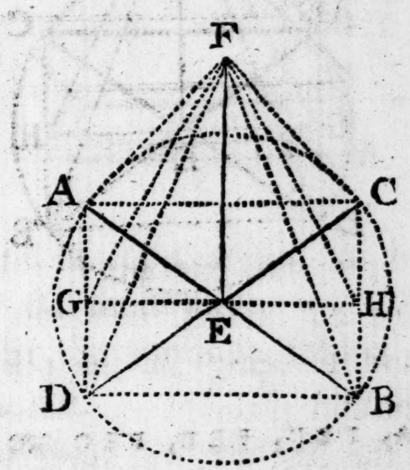
Let the plains A B, C D cut one another in E and F, and join E F; then the ſtrait line E F is their common ſection.

For the points E and F being in the plain A B, the ſtrait line E F will agree with the ſaid plain ——— ——— 7

By the ſame reaſon, the line E F will agree with the plain C D. —

Therefore, the ſtrait line E F muſt be their common ſection.

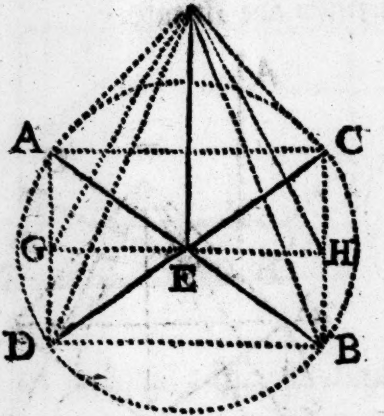
242. If a ſtrait line ſtands at right angles, to two ſtrait lines cutting one another, at the point of ſection; it will be perpendicular to the plain in which thoſe lines are ſituate.



Let the line EF ſtand at right angles, to the lines AB, CD cutting each other in E; then EF is perpendicular to the plain ACBD.

For take EA, EC, EB, ED equal to one another; draw GEH at pleaſure, joining AD, BC, and alſo FA, FG, FD, FB, FH, FC.

Now EA, ED, EB, EC are equal —— —— con.
And the contained angles AED, BEC are equal —— 69
Th. the angle GAE = EBH, and AD = BC —— 62
But the angle AEG = BEH —— —— 69
Therefore —— AG = BH, and GE = EH —— 76



2. The angles FEA, FED, FEB, FEC are equal —— 54

Th. FA, FD, FB, FC are equal —— —— 62

Th. the angles FAD, FBC are equal —— —— 65

3. The ſides FA, AG are equal to FB, BH each to each. above

And the contained angles FAG, FBH are equal —— above

Therefore, FG is equal to FH —— —— 62

Laſtly, FG, GE are equal to FH, HE each to each — above

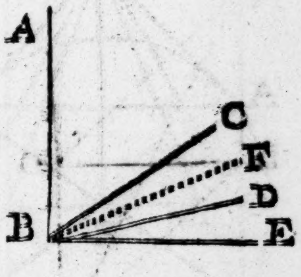
And the ſide EF is common —— —— ——

Th. the angles GEF, FEH are equal —— —— 65

Th. GEF, FEH are right angles —— —— 9

Th. EF is perpendicular to the plain ACBD —— 221

243. If a ſtrait line ſtands at right angles, to three ſtrait lines meeting one another, in their common ſection; thoſe three lines are in the ſame plain.

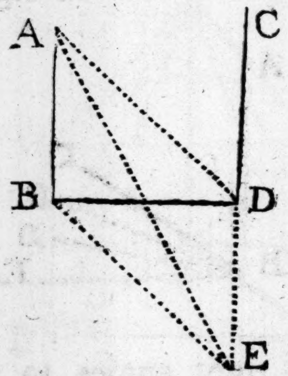


Let the line AB ſtand at right angles to the lines BC, BD, BE; then thoſe lines are in the ſame plain.

For if not, let BF be the common ſection of the plains paſſing thro' AB, BC and EB, BD.

Now BF is a ſtrait line ——— ——— 241
And AB is perpendicular to the plain paſſing thro' EB, BD. 242
Th. ABF is a right angle ——— ——— 221
But ABC is a right angle ——— ——— theo.
Th. the angles ABF, ABC are equal ——— ——— 54
Which is impoſſible ——— ——— 52
Th. BC, BD, BE muſt be in the ſame plain ——— ———

244. Two ſtrait lines perpendicular to the ſame plain, are parallels.

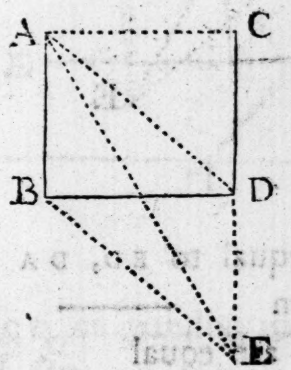


Let the lines AB, CD ſtand at right angles to the plains BD; then AB, CD are parallels.

For join BD, draw DE at right angles to BD in the given plain, make DE equal to AB, and join BE, EA, AD.

Now AB, BD are equal to ED, DB each to each ——— con.
And the contained angles ABD, EDB are equal ——— 54
Th. AD is equal to BE ——— ——— 62
Ag. AB, BE are equal to ED, DA each to each ——— above
And AE is common ——— ——— ———
Th. the angles ABE, EDA are equal ——— ——— 65
But ABE is a right angle ——— ——— 221
Th. EDA is a right angle ——— ——— 54
Laſtly, EDC is a right angle ——— ——— 221
Th. CD is in the ſame plain with DA, DB ——— 243
Alſo AB is in the ſame plain with DA, DB ——— 240
Th. AB, CD are parallels ——— ——— 79

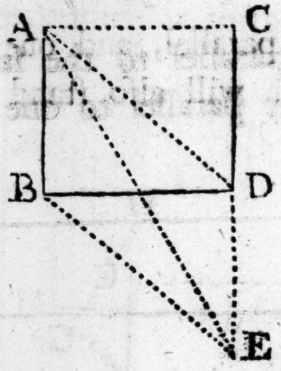
245. If two ſtrait lines be parallel, and one of them ſtands at right angles to a plain, the other will alſo ſtand at right angles to the ſame plain.



Let the lines AB, CD be parallels, and let AB ſtand at right angles to the plain BD; then CD will alſo ſtand at right angles to that plain.

For join BD, draw DE at right angles to BD in the given plain, make DE equal to AB, and join BE, EA, AD.

Now the ſides AB, BD are equal to ED, DB each to each. con.
And the contained angles ABD, EDB are equal —— 54
Th. AD is equal to BE —— —— 62.



Ag. the ſides A B, B E are equal to E D, D A each to each. above

And the ſide A E is common ——— ——— ———

Th. the angles A B E, E D A are equal ——— 65

But A B E is a right angle ——— ——— 221

Th. E D A is a right angle ——— ——— 54

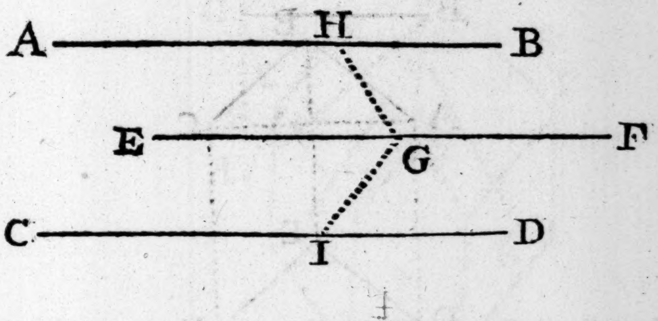
Th. D E is perpendicular to the plain A B D C, paſſing thro' B D, D A ——— ——— ——— 242

Th. C D E is a right angle ——— ——— 221

Alſo C D B is a right angle ——— ——— 82

Th. C D ſtands at right angles to the given plain, which paſſes thro' B D, D E ——— ——— ——— 242

246. Lines which are parallel to the ſame line, tho' not in the ſame plain with it; are yet parallel to one another.



Let the lines AB, CD be parallel to the line EF, but not in the ſame plain with it; I ſay, AB, CD are parallels.

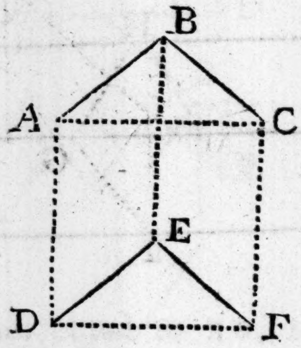
For let GH, GI be drawn at right angles to EF, and meeting AB, CD in H and I ——— ——— p. 6

Now EF is at right angles to the plain paſſing thro' GH, GI ——— ——— ——— 242

Th. AB, CD are at right angles to the ſame ——— 245

Th. AB, CD are parallels ——— ——— ——— 244

247. If two lines meeting each other, be parallel to two other lines meeting each other, tho' not in the ſame plain; yet thoſe lines will contain equal angles.

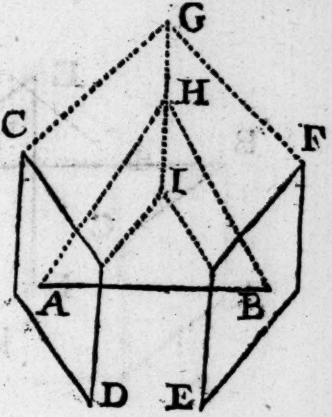


Let the lines A B, B C be parallel to D E, E F, but not in the ſame plain; I ſay, the angles A B C, D E F are equal.

For take B A, B C, E D, E F equal to one another, and join A C, D F, A D, B E, C F.

Now both A D and C F, are parallel to B E and equal to it. 88
Th. A D, C F are equal ——— ——— 48
And A D, C F are parallel ——— ——— 246
Th. A C is equal to D F —— —— —— 88
Th. A B, B C, C A, are equal to D E, E F, F D each to each. —
Th. the angle A B C is equal to D E F ——— ——— 65

248. Plains to which the ſame ſtrait line is perpendicular, are parallel to one another.

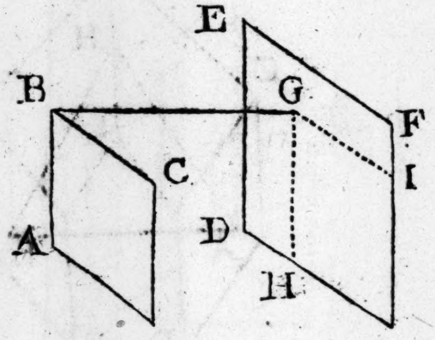


Let the ſtrait line AB, be perpendicular to the plains CD, EF; then the plains CD, EF are parallel.

For if not, let them be produced and meet in GHI, and join AH, HB.

Now ABH is a right angle	221
And BAH is a right angle	221
Therefore, the triangle ABH has two right angles	
Which is impoſſible	84
Th. the plains CD, EF produced, will not meet	
Th. the plains CD, EF are parallel	225

249. If two lines meeting each other, be parallel to two other lines meeting each other, but not in the ſame plain; the plains which paſs thro' them, will be parallel.

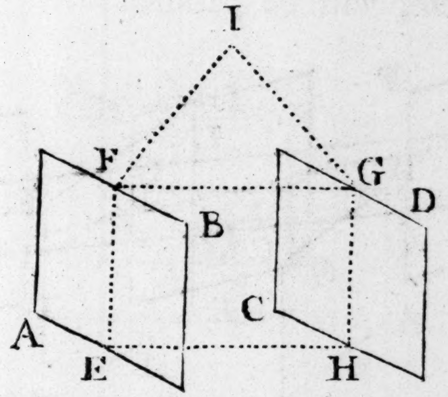


Let the lines AB, BC be parallel to DE, EF not in the ſame plain; I ſay, the plains paſſing thro' AB, BC and DE, EF are parallel.

For let BG be drawn perpendicular to the plain DF, and GH, GI parallel to ED, EF.

Now BGH is a right angle ——— ——— 221
And BA is parallel to GH ——— ——— ——— 246
Th. GBA is a right angle ——— ——— ——— 82
By the ſame reaſon GBC is a right angle ——— ———
Th. BG is perpendicular to the plain AC ——— 242
Th. the plains AC, DF are parallel ——— ——— 248

250. If a plain cuts two parallel plains, their common ſections will be parallel.



Let the parallel plains A B, C D be cut by the plain E F G H, making the ſections E F, G H; then the lines E F, G H are parallels.

For if not, let them be produced and meet in I. 59

Now E F, H G are ſtrait lines —— —— 241

And each line agrees with its plain —— 7

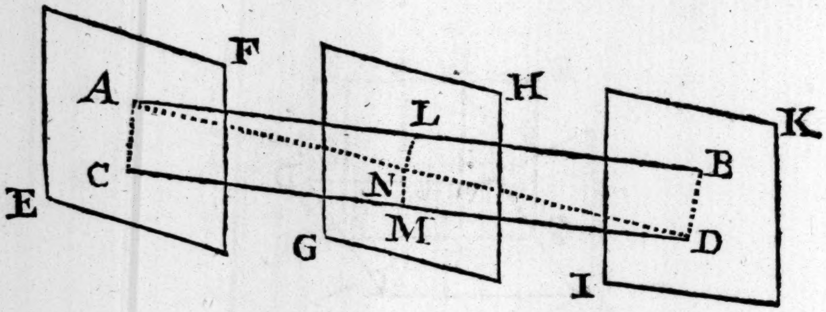
Th. the plains A B, C D being produced, will meet in I ——

Which is impoſſible —— —— —— 225

Th. the lines E F, H G being produced, will not meet

Th. E F, G H are parallels —— —— 12

251. If two ſtrait lines are cut by parallel plains, they will be cut proportionally.

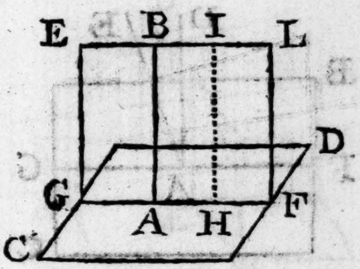


Let the ſtrait lines AB, CD be cut by parallel plains EF, GH, IK in the points A, L, B, C, M, D; then AB : AL :: CD : CM.

For join AC, AD, DB, let AD cut the plain GH in N, and draw LN, NM.

Now the triangle ABD lies in a plain —— 240
Th. LN and BD are parallel —— —— 250
By the ſame reaſon, NM and AC are parallel —— ——
Th. AN : ND :: AL : LB —— —— 185
And AN : ND :: CM : MD —— —— 185
Th. AL : LB :: CM : MD —— —— 169
Th. AB : AL :: CD : CM —— —— 172

252. If a ſtrait line ſtands at right angles to any plain, all the plains which paſs thro' it will be perpendicular to the ſame plain.



Let the line A B ſtand at right angles to the plain C D, and let the plain E F paſs thro' the ſaid line A B; then the plain E F is perpendicular to the plain C D.

For let F G be the common ſection of thoſe plains, in which aſſume the point H, and in the plain E F erect H I perpendicular to G F. p. 6

Now B A H is a right angle — — — theo.

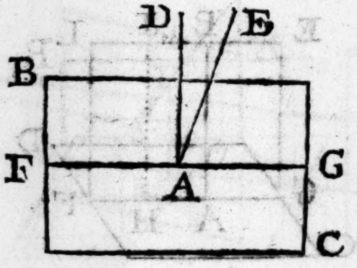
And A H I is a right angle — — — con.

Th. A B, H I are parallel — — — — 79

Th. H I is at right angles to the plain C D — — 245

Th. the plain E F is perpendicular to the plain C D — 222.

253. At a given point in a given plain, only one perpendicular can be erected to that plain, on the ſame ſide.



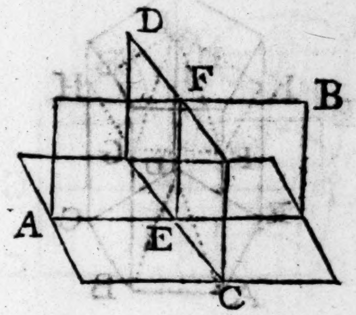
At the point A, in the plain B C; only one perpendicular can be erected to that plain.

For if poſſible, let both A D and A E be perpendicular to the plain B C, and thro' D A, A E let a plain be drawn, making with B C the ſection F A G.

Now F A G is a ſtrait line —— —— —— 241
Th. D A F is a right angle —— —— —— 221
Alſo E A F is a right angle —— —— —— 221
Th. the angles D A F, E A F are equal —— —— 54
Which is impoſſible —— —— —— 52

Conſequently, only one perpendicular can be erected to a given plain, at a given point in it.

254. If two plains cutting each other, be perpendicular to ſome plain, their common ſection will alſo be perpendicular to that plain.



Let the plains A B, C D cutting each other, be perpendicular to the plain A E; then their common ſection E F will alſo be perpendicular to the ſame plain.

For let A E, E C be the common ſections made with the plain A E, and at the point E to each of the lines A E, E C, in each of the plains A B, C D erect a perpendicular ——— ——— p. 6

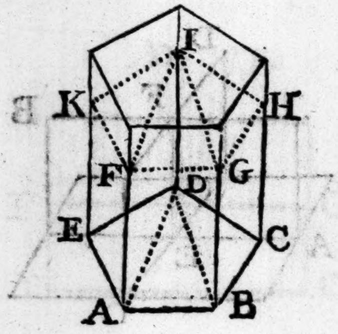
Now the perpendiculars erected to A E, E C in the plains A B, C D, are perpendicular to the plain A E ——— 222

But two perpendiculars cannot be erected to a plain, at the ſame point and on the ſame ſide ——— ——— 253

Conſequently, both thoſe perpendiculars muſt agree with E F, the common ſection of the plains A B, C D ——— 241

Th. E F is perpendicular to the plain A E ——— ———

255. If a priſm be cut by a plain parallel to its baſe, the ſection will be equal to the baſe.



Let ABCDE be the baſe of a priſm, and FGHIK a ſection parallel to it; then FGHIK is equal to ABCDE.

For join AD, DB, FI, IG.

Now DI is parallel to AF, and to BG ——— 246

Th. FI, IG are parallel to AD, DB ——— 250

Th. FI, IG are equal to AD, DB ——— ——— 89

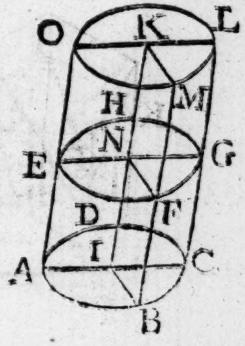
And the contained angles FIG, ADB are equal ——— 247

Th. the triangle FIG is equal to ADB ——— 62

By the ſame reaſon, each triangle in the ſection, is equal to its correſponding triangle in the baſe. ——— ——— ———

Th. FGHIK is equal to ABCDE ——— ——— 49

256. If a cylinder be cut by a plain parallel to its baſe, the ſection will be a circle, equal to the baſe.

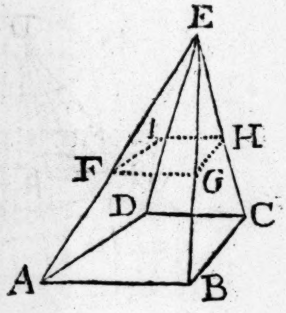


Let ABCD be the baſe of a cylinder AL, and EFGH be a ſection parallel to the baſe; then EFGH will be a circle, and equal to ABCD.

For let the plains IKLC, IKMB cut the cylinder thro' its axis IK, and meet the ſection in N, F, G.

Now BM, CL are parallel to IK —— —— 234
And NF, NG are parallel to IB, IC —— —— 250
Th. NF, NG are equal to IB, IC —— —— 89
But IB and IC are equal —— —— —— 30
Th. NF and NG are equal —— —— —— 48
By the ſame reaſon, all lines drawn from the point N to the circumference of the ſection, are equal —— ——
Th. the ſection EFGH is a circle —— —— 30
And the circles EFGH, ABCD are equal —— 55

257. If a pyramid be cut by a plain parallel to its baſe, the ſection will be ſimilar to the baſe.



Let ABCDE be a pyramid, and FGHI a ſection parallel to the baſe; then FGHI is ſimilar to ABCD.

For FG, GH are parallel to AB, BC —— —— 250

Th. EB : EG :: AB : FG —— —— —— 189

And EB : EG :: BC : GH —— —— —— 189

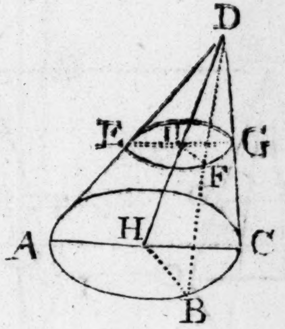
Th. AB : FG :: : GH —— —— —— 169

And the angles ABC, FGH are equal —— 247

By the ſame reaſon, each angle in the ſection is equal to its correſponding angle in the baſe, and the ſides about them are proportional —— —— —— ——

Th. FGHI is ſimilar to ABCD —— —— 161

258. If a cone be cut by a plain parallel to its baſe, the ſection will be a circle.



Let ABCD be a cone, and EFG a ſection parallel to the baſe; then EFG is a circle.

For let the plains ADC, BDH cut the cone thro' its axis DH, and meet the ſection in I, F, G.

Now IF, IG are parallel to HB, HC —— —— 250

Th. DH : DI :: HB : IF —— —— —— 189

And DH : DI :: HC : IG —— —— —— 189

Th. HB : IF :: HC : IG —— —— —— 169

But HB is equal to HC —— —— —— 30

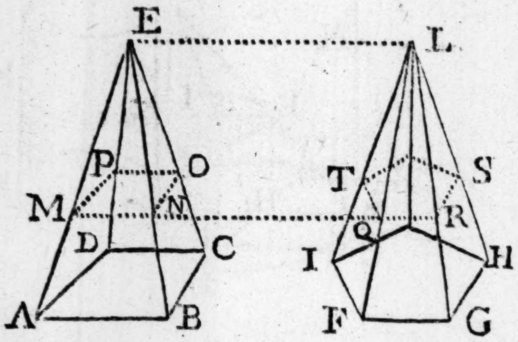
Th. IF is equal to IG —— —— —— 163

By the ſame reaſon, all lines drawn from the point I to the circumference of the ſection, are equal to IG.

Th. the ſection EFG is a circle —— —— 30

259. If

259. If pyramids or cones of equal altitude ſtanding upon the ſame plain, be cut by a plain parallel to their baſes; the ſections and baſes will be proportional.



Let the pyramids ABCDE, FGHIL of equal altitude, ſtanding upon the ſame plain, be cut by a plain parallel to their baſes AC, FH, making the ſections MNOP, QRST; then AC : MO :: FH : QS.

For MN, QR are parallel to AB, FG ——— 250

Th. EA : EM :: AB : MN ——— ——— 189

And LF : LQ :: FG : QR ——— ——— 189

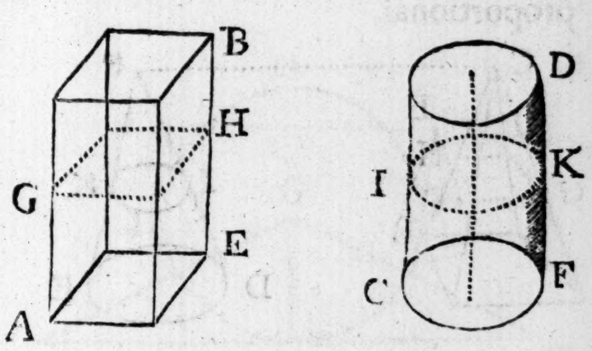
But EA : EM :: LF : LQ ——— ——— 251

Th. AB : MN :: FG : QR ——— ——— 169

Th. $AB^2 : MN^2 :: FG^2 : QR^2$ ——— ——— 203

Th. AC : MO :: FH : QS ——— ——— 208

260. Priſms and cylinders, of equal baſe and altitude, are equal to one another.

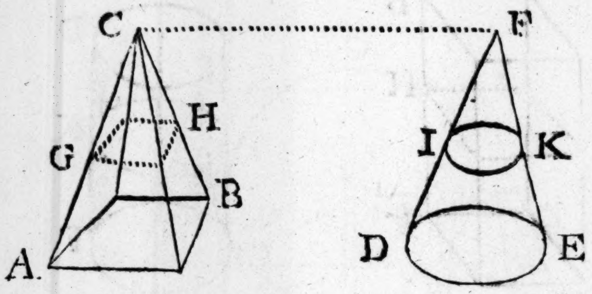


Let the priſm A B and cylinder C D, have equal baſes A E, C F, and alſo equal altitudes; then the ſolids A B, C D are equal.

For conceive them to ſtand upon the ſame plain, and to be cut by plains parallel to their baſes, making the correſponding ſections G H, I K, &c.

Now the ſection G H is equal to the baſe A E —— 255
And the ſection I K is equal to the baſe C F —— 256
But the baſes A E, C F are equal —— —— theo.
Th. the ſections G H, I K are equal —— —— 48
By the ſame reaſon, every ſection in A B is equal to its correſponding ſection in C D —— —— —— ——
Th. the ſolids A B, C D are equal —— —— 239

261. **Pyramids and cones of equal baſe and altitude, are equal to one another.**



Let the pyramids or cones ABC, DEF have equal baſes AB, DE, and alſo equal altitudes; then the ſolids ABC, DEF are equal.

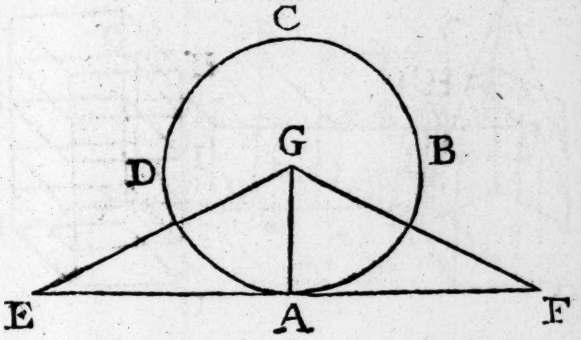
For let them ſtand upon the ſame plain, and be cut by plains parallel to their baſes, making the correſponding ſections GH, IK, &c.

Now AB : GH :: DE : IK ——— ——— 259
And AB is equal to DE ——— ——— theo.
Th. GH is equal to IK ——— ——— 163

By the ſame reaſon, every ſection made in ABC, is equal to the correſponding ſection made in DEF ——— ———

Th. the ſolids ABC, DEF are equal ——— 239

262. A globe is equal to a pyramid or cone, whoſe baſe is equal to its ſurface, and altitude equal to its ſemidiameter.



Let the ſurface of the globe A B C D, be equal to the baſe of the pyramid E F G, whoſe altitude is the ſemidiameter of the globe G A; then the globe is equal to the pyramid.

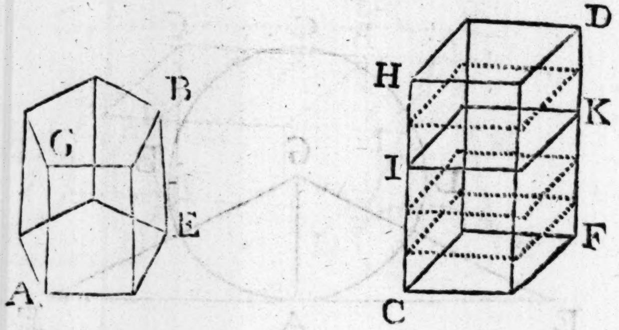
For let the ſurface of the globe and baſe of the pyramid, be divided into indefinitely ſmall parts equal to one another, upon which as baſes let pyramids be conſtituted, whoſe common vertex is in G, the center of the globe ——— ——— 168

Now the number of baſes in the ſurface of the globe, are equal to the number in the baſe in the pyramid —— 51

And the pyramids conſtituted thereon are equal —— 261

Th. the globe A B C D is equal to the pyramid E F G — 49

263. Priſms and cylinders that have equal baſes, are proportional to their altitudes.



Let AB, CD be priſms or cylinders having equal baſes AE, CF, but unequal altitudes AG, CH; then AG : CH :: AB : CD.

For in CH the greater altitude, take CI equal to AG the leſſer, conceive CI, IH divided into parts equal to one another, and plains paſſing thro' each point of diviſion parallel to the baſe CF.

Now each ſection in CD, is equal to the baſe CF —— 255

Th. thoſe ſections are equal to one another. —— 48

Th. all the little ſolids into which CD is divided by the cutting plains, are equal to one another. —— —— 260

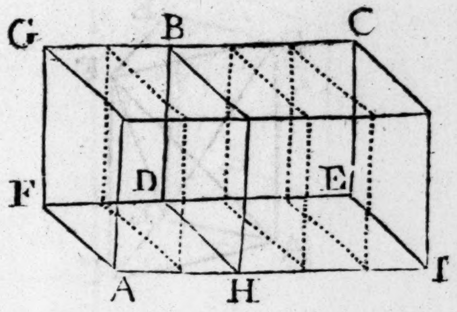
Th. CI, CH and CK, CD have equal ratios —— 155

Th. CI : CH :: CK : CD —— —— —— 156

But AB is equal to CK —— —— —— 260

Th. AG : CH :: AB : CD —— —— —— 56

264. Priſms and cylinders having equal altitudes, are proportional to their baſes.

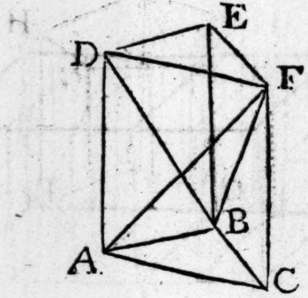


Since all priſms and cylinders of equal baſe and altitude are equal (by 260) let the rectangular priſms AB, AC, whoſe baſes are AD, AE, repreſent any priſms or cylinders of equal altitude FG; then AD : AE :: AB : AC.

For conceive FD, DE divided into parts equal to one another, and plains drawn thro' each point of diviſion parallel to AG or BH.

Now each ſection in the ſolid AC is equal to AG —— 255
Th. the ſections are equal to one another —— 48
And the ſections in AE are parallel —— —— 250
Th. the little rectangles in AD, AE are equal —— 92
And the little priſms in AB, AC are equal —— 260
Th. AD, AE and AB, AC have equal ratios —— 155
Th. AD : AE :: AB : AC —— —— 156

265. Every pyramid that has a triangular baſe, is a third part of a priſm having the ſame baſe and altitude.

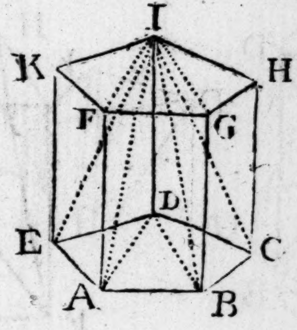


Let the triangle ABC be the baſe of a pyramid ABCF, and a priſm having the ſame altitude; then the pyramid is a third part of the priſm.

For let DEF be the oppoſite baſe of the priſm, and join AF, FB, BD.

Now the baſe ACF is equal to the baſe ADF —— 89
And the vertex B is common —— —— ——
Th. the pyramids ACFB, ADFB are equal —— 261
Again, the baſe ADB is equal to the baſe BDE —— 89
And the vertex F is common —— —— ——
Th. the pyramids ADBF, BDEF are equal —— 261
But the pyramid ADBF is the ſame with ADFB, being contained by the very ſame plains —— —— ——
Th. the pyramids ACFB, ADFB, BDEF are equal — 48
And the priſm ABCDEF is equal to thoſe three pyramids. 52
Th. ACFB, or ABCF is one third of ABCDEF —— ——

266. Every pyramid is a third part of a priſm, having the ſame baſe and altitude.



Let ABCDE be the common baſe of a pyramid ABCDEI, and a priſm AHIK; then the pyramid is a third part of the priſm.

For join AD, DB, FI, IG.

Now upon each triangle in the baſe ABCDE, is conſtituted a pyramid and a priſm, of the ſame altitude ——— con.

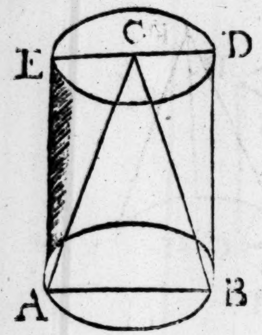
And each of thoſe pyramids is a third part of each priſm, having the ſame baſe —— —— —— 265

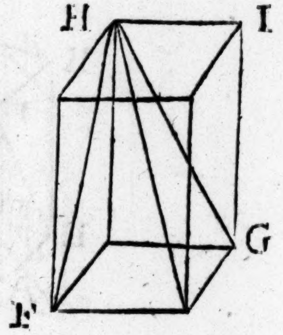
Therefore, all the pyramids taken together, are a third part of all the priſms taken together —— —— ——

But all thoſe pyramids compoſe the pyramid ABCDEI, and all theſe priſms compoſe the priſm AHIK —— —— ——

Th. ABCDEI is one third of AHIK —— —— 56

267. Every cone is a third part of a cylinder, having the ſame baſe and altitude.





If there be a cone ABC, and a cylinder ABDE having the ſame baſe and altitude; then the cone is one third of the cylinder.

For let there be a pyramid FGH, and a priſm FGIH, having an equal baſe and altitude with the cone and cylinder ——

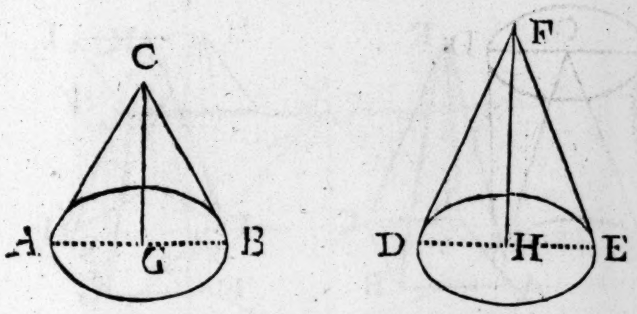
Now the priſm FI, is equal to the cylinder AD —— 260

And the pyramid FGH, is equal to the cone ABC —— 261

But the pyramid FGH, is one third of the priſm FI — 266

Th. the cone ABC, is one third of the cylinder AD — 56

268. Pyramids and cones having equal baſes, are proportional to their altitudes.



Let ABC, DEF be pyramids or cones, with equal baſes, but unequal altitudes CG, FH; then CG : FH :: ABC : DEF.

For priſms or cylinders having equal baſes, are proportional to their altitudes ——— ——— ——— 263

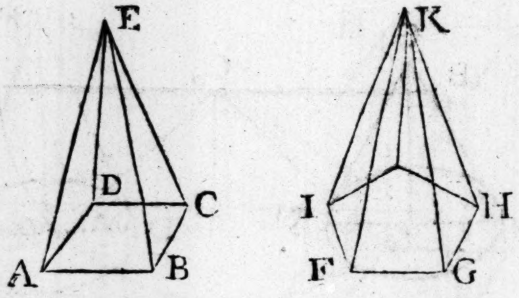
Namely, CG : FH :: cyl. : cyl. ——— ——— 263

But quantities and their third parts are proportional ——— 170

And pyramids or cones ABC, DEF are the third parts of priſms or cylinders, of equal baſe and altitude ——— 267

Th. CG : FH :: ABC : DEF ——— ——— 169

269. Pyramids and cones of equal altitude, are proportional to their baſes.



Let ABCDE, FGHIK be pyramids or cones of equal altitude; then as the baſe AC to the baſe FH, ſo is the ſolid E to the ſolid K.

For priſms or cylinders of equal altitude, are proportional to their baſes —— —— —— —— 264

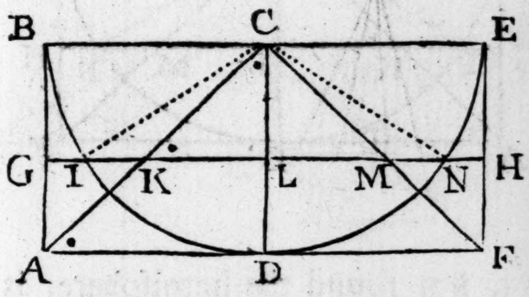
Namely AC : FH :: priſm : priſm —— —— 264

But quantities and their third parts are proportional —— 170

And pyramids or cones E, K, are the third parts of priſms or cylinders, of equal baſe and altitude —— —— 267

Th. AC : FH :: ABCDE : FGHIK —— —— 169

270. A hemiſphere, is two thirds of its circumſcribing cylinder.



Let the ſquare ABCD, the triangle ACD, and the quadrant BCD, revolve about CD as an axis, and generate the cylinder ABEF, the cone AFC, and the hemiſphere BDE; then $BDE = \frac{2}{3} ABEF$.

For let GH be a ſection parallel to the baſe of the cylinder, and AE another ſection thro' its axis, cutting the former in I, K, L, M, N.

Now it is evident by the conſtruction of the figure, that CI, CD, DA, GL, LH are equal —— —— ——

Alſo, that $IL = LN$, $KL = LM$, $KL = LC$, and the angles at the point L are right angles —— —— ——

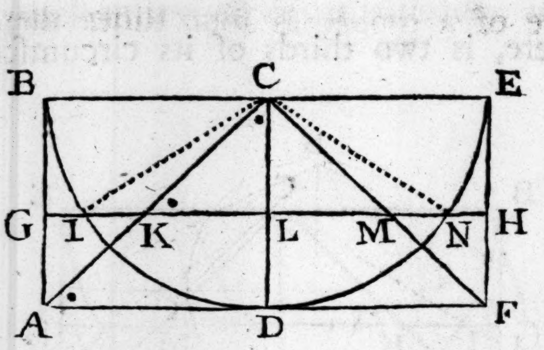
Th. $CI^2 = IL^2 + LC^2$ —— —— 98

Th. $GL^2 = IL^2 + KL^2$ —— —— 56

Th. $4GL^2 = 4IL^2 + 4KL^2$ —— —— 49

Or $GH^2 = IN^2 + KM^2$ —— —— 108

Th. circle GH = circle IN + circle KM —— 220



Th. the ring GI, NH round the hemiſphere, is equal to the circle whoſe diameter is KM ——— ——— ——— 50

By the ſame reaſon, every ſection made in the cup like ſolid ABIDNEF, is equal to its correſponding ſection in the cone AFC.

Th. ABIDNEF is equal to the cone AFC ——— 239

But the cone AFC, is one third of the cylinder ABEF 267

Therefore, the ſolid ABIDNEF, is one third of the cylinder ABEF ——— ——— ——— ——— 56

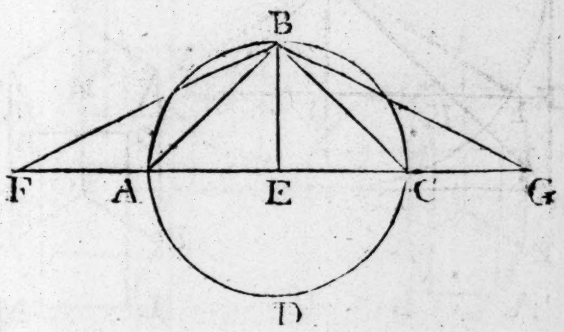
Th. the hemiſphere BDE $= \frac{2}{3}$ ABEF ——— ——— 50

Corollary.

271. A cone, globe, and cylinder circumſcribing them, are proportional to the numbers 1, 2, 3.

272. A globe is four times a cone, whoſe baſe is its great circle, and altitude its ſemidiameter.

273. The ſurface of a globe, is four times the area of its great circle.

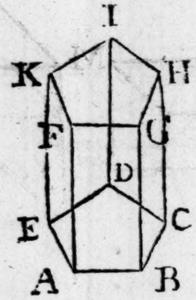


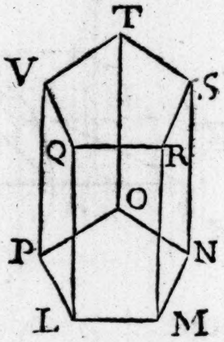
Let the globe A B C D be cut by the plains A B C D, F A C G paſſing through the center E, perpendicular to each other, whoſe common ſection A C is a diameter of the globe : then the ſurface of the globe is four times the circle whoſe diameter is A C.

For let A C B, F G B be cones, whoſe common altitude is the ſemidiameter E B, and let the baſe of F G of the greater be equal to the ſurface of the globe.

Now the globe is equal to the cone F G B —— 262
And the globe is four times the cone A C B —— 272
Th. the cone F G B is four times the cone A C B —— 56
Th. the baſe F G is four times the baſe A C —— 269
But the baſe F G is equal to the ſurface of the globe — con.
Th. the ſurface of the globe, is four times the area of its great circle, whoſe diameter is A C —— —— 56

274. The ſurfaces of ſimilar priſms, are proportional to the ſquares of their correſponding lines.

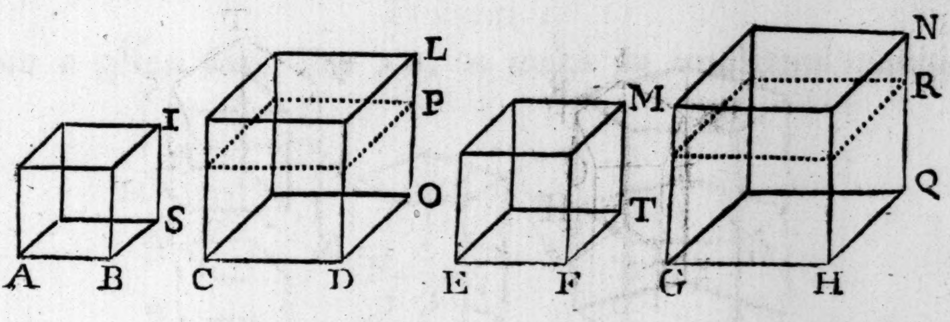




Let AH, LS be ſimilar priſms, whoſe baſes are AC, LN, and AB, LM correſponding lines; then AB^2 : LM^2 :: ſurface of AH : ſurface of LS.

For AB : LM :: BC : MN :: CD : NO, &c. —— 161
Th. AB^2 : LM^2 :: BC^2 : MN^2 :: CD^2 : NO^2, &c. —— 203
But AB^2 : LM^2 :: AC : LN :: AG : LR, &c. —— 208
Th. AC : LN :: AG : LR :: BH : MS, &c. —— 169
Th. AC : LN :: ſurf. AH : ſurf. LS —— —— 175
Th. AB^2 : LM^2 :: ſurf. AH : ſurf. LS —— 169

275. If ſtrait lines are proportional, their cubes will alſo be proportional.



Let AB, CD, EF, GH be proportional lines, whoſe cubes are AI, CL, EM, GN and AB : CD :: EF : GH; then AI : CL :: EM : GN.

For in LO, NQ take OP, QR equal to AB, EF or SI, TM each to each, and through the points PQ, let plains cut thoſe cubes parallel to their baſes CO, GQ.

Now AB : CD :: EF : GH —— —— theo.

Th. AS : CO :: ET : GQ —— —— 203

But AS : CO :: AI : CP —— —— 264

And ET : GQ :: EM : GR —— —— 264

Th. AI : EM :: CP : GR* —— —— 169

Ag. OP : OL :: CP : CL —— —— 263

And QR : QN :: GR : GN —— —— 263

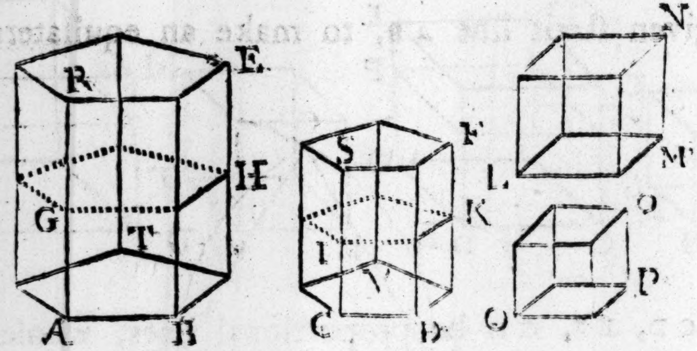
But OP : OL :: QR : QN by article 56, being equal to AB, CD, EF, GH each to each —— —— con.

Th. CP : GR :: CL : GN* —— —— 169

Th. AI : EM :: CL : GN —— —— 169

Th. AI : CL :: EM : GN —— —— 171

275*. Similar priſms, are proportional to the cubes of their correſponding lines.



Let AE, CF be ſimilar priſms, whoſe baſes are AT, CV, and AB, CD correſponding lines; then, as cube of AB : cube of CD :: AE : CF.

For make AG, CI equal to AB, CD, let GH, IK be ſections parallel to the baſes, and LN, OQ be the cubes of AB, CD.

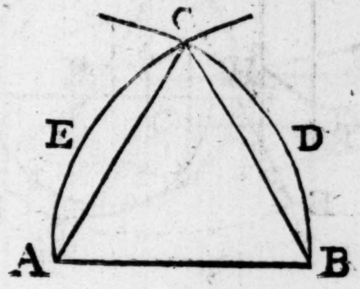
Now AR : AG or AB :: AE : AH —— —— 263
And CS : CI or CD :: CF : CK —— —— 263
But AR : AB —— :: CS : CD —— —— 238
Th. AE : CF —— :: AH : CK * —— —— 169
Ag. LM or AB^2 : AT :: LN : AH —— —— 264
And OP or CD^2 : CV :: OQ : CK —— —— 264
But AB^2 : AT —— :: CD^2 : CV —— —— 208
Th. LN : OQ —— :: AH : CK* —— —— 169
Th. LN : OQ —— :: AE : CF —— —— 169
Or cube of AB : cube of CD :: AE : CF —— ——

The end of the theorems.

Pro-

Problem I.

Upon a given ſtrait line A B, to make an equilateral triangle. 1. E. 1.



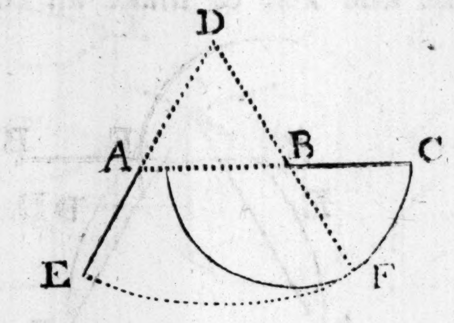
About the center A with the radius A B deſcribe the circle B D C by article 60, alſo about the center B with the radius B A deſcribe the circle A E C, and join A C, C B.

For A C is equal to A B — — — 30
And B C is equal to B A — — — 30
Th. A C is equal to B C — — — 48
Th. A B, B C, C A are equal — — — 48
Th. the triangle A B C is equilateral — — 15

Pro-

Problem 2.

2. l. 1. At a given point A, to put the end of a line that ſhall be equal to a given line B C.



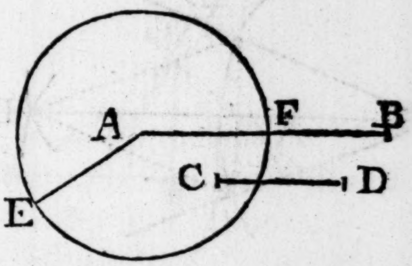
Join A B, and upon it make the equilateral triangle A B D, produce the ſides D A, D B, to E and F, about the center B, with the radius B C deſcribe the arch C F, and about the center D with the radius D F, deſcribe the arch F E.

For D E is equal to D F ——— ——— 30
And D A is equal to D B ——— ——— con.
Th. A E is equal to B F ——— ——— 50
But B C is equal to B F ——— ——— 30
Th. A E is equal to B C ——— ——— 48

Pro-

Problem 3.

Two unequal lines AB and CD being given, to take a part of 3.b.1. the greater that ſhall be equal to the leſſer.



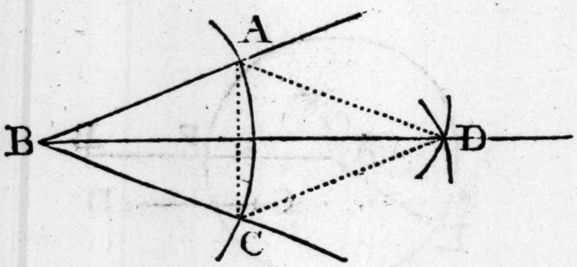
At the point A put the line AE equal to the line CD by problem 2, and with the radius AE deſcribe a circle cutting AB in F.

For AE is equal to AF —— —— 30
And AE is equal to CD —— —— con.
Th. AF is equal to CD —— —— 48

Pro-

Problem 4.

9. 6. 1. To divide a given angle ABC into two equal parts, or to biſect it.



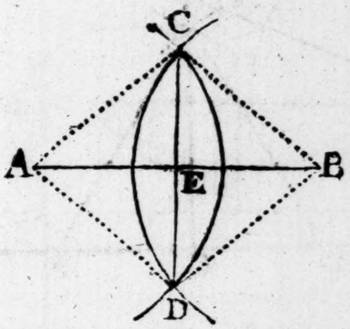
On the center B at any diſtance BA deſcribe the arch AC, on the centers A and C with the diſtances AB, BC deſcribe arches cutting each other in D, and draw BD.

For join AD, DC, CA.

Now AB, BC, CD, DA are equal ——— ——— con.
Th. the angle BAC = BCA ——— ——— 63
And the angle DAC = DCA ——— ——— 63
Th. the angle BAD = BCD ——— ——— 49
Th. the angle ABD = DBC ——— ——— 62

Problem 5.

To divide a given finite right line A B, into two equal parts, or to biſect it. 10. E. 1.



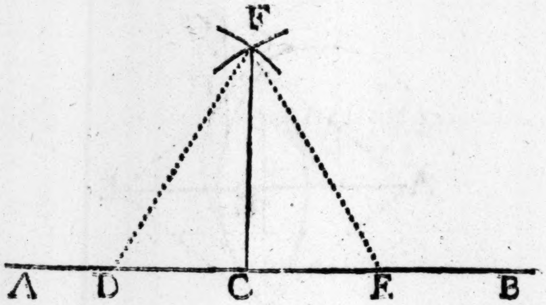
About the centers A and B, with any radius greater than half the line, deſcribe circles cutting one another in C and D, and draw C D cutting A B in E.

For join A C, C B, B D, D A.

Now A C, C B, B D, D A are equal —— —— con.
Th. the angle C A B is equal to C B A —— 63
And the angle D A B is equal to D B A —— 63
Th. the angle C A D is equal to C B D —— 49
Th. the angle A C D is equal to B C D —— 62
Or the angle A C E is equal to B C E —— ——
Th. A E is equal to E B —— —— 62

Problem 6.

11. 6. 6. To erect a perpendicular to a given right line AB, at a given point in it C.



About the center C, with any diſtance deſcribe a circle cutting the given line in D and E, about the centers D and E, with any diſtance greater than CD deſcribe circles cutting in F, and draw CF.

For join DF, FE.

Now CD is equal to CE —— —— 30

—— DF is equal to FE —— —— con.

And CF is common —— —— —— ——

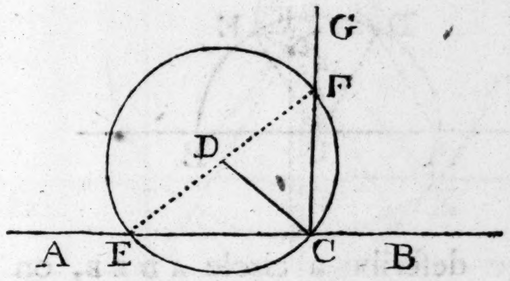
Th. the angle DCF = FCE —— —— 65

Th. CF is perpendicular to AB —— —— 9

6. Second

6. Second method.

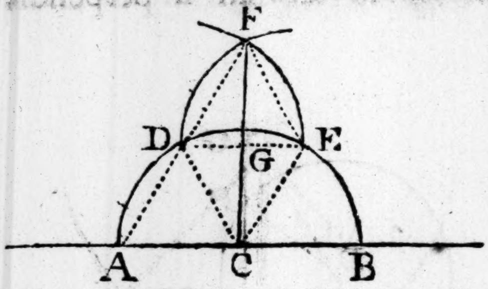
To erect a perpendicular to a given line AB, at a given point in it C.



Aſſume the center D, and with the radius DC deſcribe the circle CEF cutting AB in E, join ED, which continue to F, and draw CFG.

For EF is a diameter of the circle —— —— 31
Th. EFC is a ſemicircle —— —— 33
Th. ECF is a right angle —— —— 149
Th. CFG is perpendicular to AB —— —— 9

6. Third method.



About the center C deſcribe a circle ADEB, on the center A cut the circle in D, on the center D deſcribe the arch FE, and on E cut that arch in F, all with the ſame radius, and draw CF.

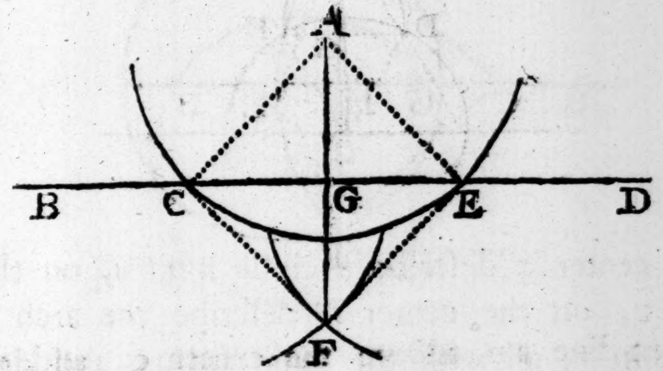
For join AD, DF, FE, EC, CD, DE.

Now the triangles ADC, CDE, EDF are equilateral —— con.
Th. the angles ACD, CDE, EDF are equal —— 65
Th. the angles DGF, DGC are equal —— —— 62
Th. DGF is a right angle —— —— —— 9
Ag. DE is parallel to AB —— —— —— 77
Th. the angle ACF is equal to DGF —— —— 81
Th. ACF is a right angle —— —— —— 54
Th. CF is perpendicular to AB —— —— 9

Pro-

Problem 7.

From a given point A, to let fall a perpendicular upon an infinite ſtrait line B D. 12. L. 1.



About the center A, deſcribe a circle at pleaſure cutting the given line in C and E, about the centers C and E with the ſame radius deſcribe circles cutting one another in F, and draw A F cutting B D in G.

For join A C, C F, F E, E A.

Now A C, C F are equal to A E, E F —— —— con.

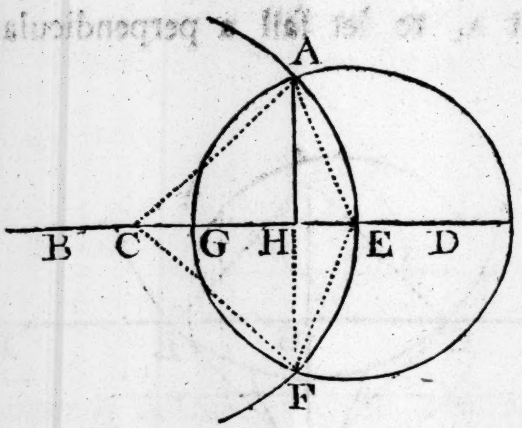
And the ſide A F is common —— —— ——

Th. the angle C A F is equal to F A E —— —— 65

Th. the angle C G A is equal to A G E —— —— 62

Th. A G is perpendicular to B D —— —— 9

7. Otherwiſe



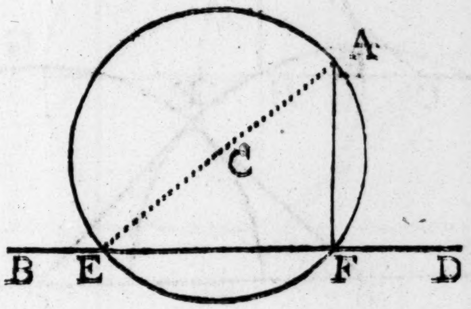
In the given line BD aſſume the center C, and with the radius CA deſcribe the circle AEF cutting the line in E, about the center E, at the diſtance EA, deſcribe the circle AGF, and draw AF cutting BD in H.

For join AC, CF, FE, EA.

Now CA is equal to CF —— —— —— 30
—— EA is equal to EF —— —— —— 30
And the ſide CE is common —— —— —— ——
Th. the angle ACE = ECF —— —— —— 65
Th. the angle AHC = CHF —— —— —— 62
Th. AH is at right angles to BD —— —— 9

7. Third method.

From a given point A, to let fall a perpendicular upon an infinite line B D.



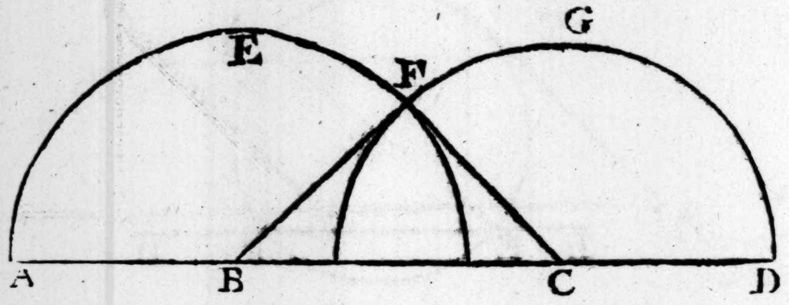
In the given line B D aſſume the point E, join A E, and biſect it in C by problem 5, about the center C with the radius C A deſcribe a circle cutting B D in F, and draw A F.

For A E F is a ſemicircle —— —— —— con.
Th. A F E is a right angle —— —— 149
Th. A F is perpendicular to B D —— —— 9

Pro

Problem 8.

22 E. 1. To make a triangle whoſe ſides ſhall be equal to three given lines AB, BC, CD; provided any two of them taken together be greater than the third.



About the center B, with the radius BA, deſcribe the circle AEF, on the center C, with the radius CD, deſcribe the circle DGF, and draw BF, FC.

For BF is equal to BA ——— ——— 30

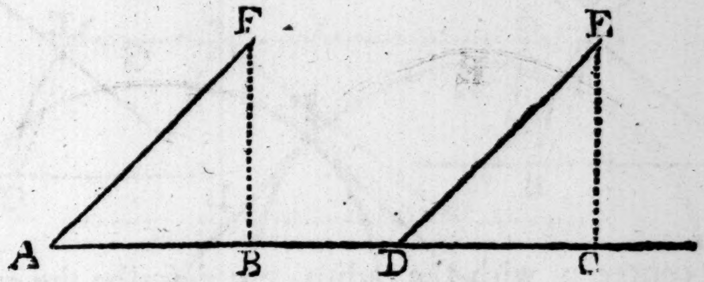
And CF is equal to CD ——— ——— 30

Th. BFC is the triangle required.

Pro-

Problem 9.

At a given point A, in a given ſtrait line AB, to make an angle equal to a given angle CDE. 23. & 1.

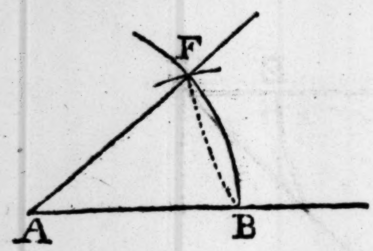


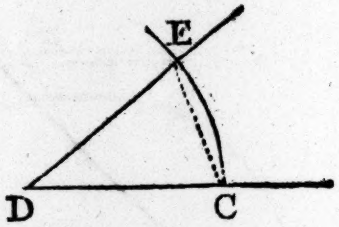
Draw CE at right angles to CD by problem 6, make AB equal to DC by problem 3, erect BF perpendicular to AB, make BF equal to CE, and draw AF.

For AB is equal to DC ——— ——— con.
---- BF is equal to CE ——— ——— con.
And the angle ABF is equal to DCE ——— 54
Th. the angle BAF is equal to CDE ——— 62

9. Otherwiſe

At a given point A, in a given ſtrait line A B, to make an angle equal to a given angle C D E.





About the center D with the radius D C deſcribe the arch C E, about the center A with the ſame radius D C deſcribe the arch B F, on the center B, with the diſtance C E cut B F in the point F, and draw A F.

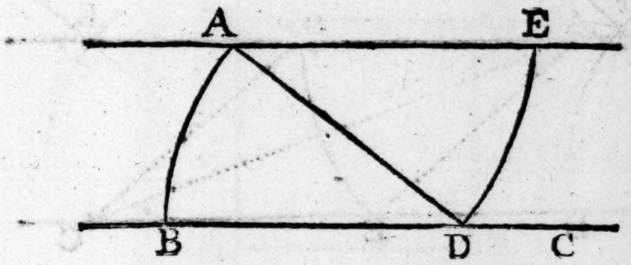
For join B F, C E.

Now A B is equal to D C ——— ——— ——— con.
----- A F is equal to D E ——— ——— ——— con.
And B F is equal to C E ——— ——— ——— con.
Th. the angle A is equal to the angle D. ——— ——— 65

Pro-

Problem 10.

Through a given point A, to draw a line parallel to a given ſtrait line B C. 31. B.1.



In the given line take the point D at pleaſure and join A D, make the angle D A E equal to the angle A D B, and the thing is done.

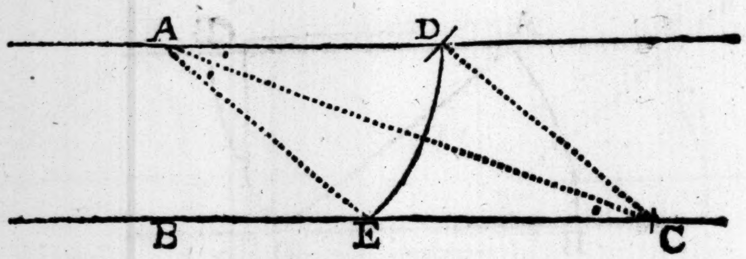
For the angle D A E = A D B ——— ——— con.

And they are alternate angles ——— ———

Th. A E is parallel to B C ——— ——— 77

10. Otherwiſe.

Through a given point A, to draw a line parallel to a given ſtrait line B C.



In the given line aſſume the point E, and about the center A with the radius A E deſcribe the arch E D, make E C equal to A E, on the center C, with the radius A E or C E, cut the arch E D in D, and draw A D.

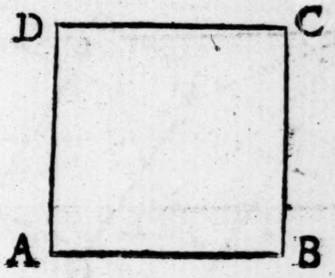
For join A E, A C, C D.

Now A D = A E, and C D = C E	30
And E C = A E	con.
Th. A D, D C, C E, E A are equal	48
And the ſide A C is common	
Th. the angle D A C = E C A	65
Th. A D is parallel to B C	77

Pro-

Problem 11.

To deſcribe, or make a ſquare upon a given ſtrait line A B, 46. E. I.



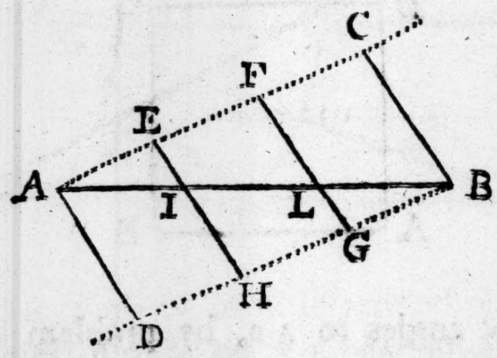
Draw A D at right angles to A B, by problem 6, make A D equal to A B, draw D C parallel to A B, and B C parallel to A D by problem 10.

For A B C D is a parallelogram	22
Th. A B = D C and A D = B C	89
Th. A B, B C, C D, D A are equal	48
And the angles are all right angles	90
Th. A B C D is a ſquare	24

Pro-

Problem 12.

To divide a given ſtrait line AB, into a propoſed number of equal parts.



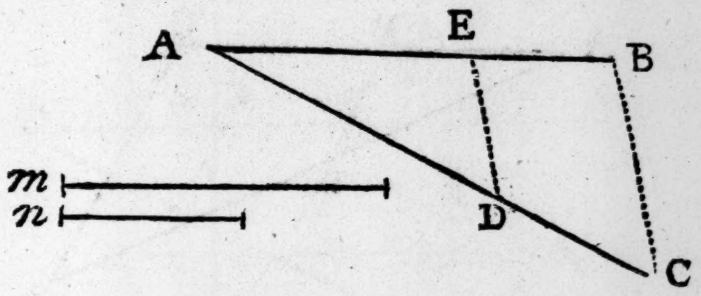
Draw AC at pleaſure, and BD parallel to AC by problem 10, in AC, BD take the propoſed number of equal parts of any length, namely, AE, EF, FC, BG, GH, HD, all equal to one another, and draw EH, FG cutting the given line in I and L.

For join BC.

Now FC, GB are equal and parallel —— —— con.
Th. FG is parallel to CB —— —— 88
By the ſame reaſon EH is parallel to CB —— ——
But AE, EF, FC are equal —— —— con.
Th. AI, IL, LB are equal —— —— 99

Problem 13.

To divide a given ſtrait line AB, in a given ratio.



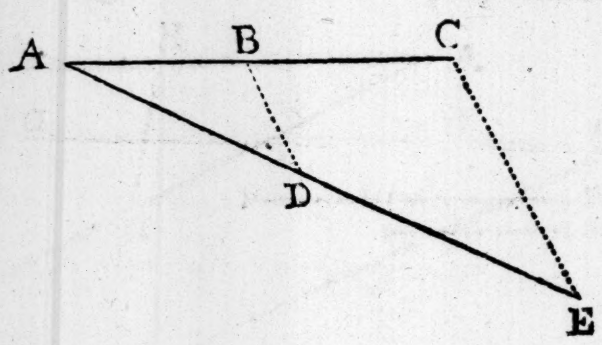
Let the given ratio be that of *m* to *n*, draw AC at pleaſure, and in it take AD equal to *m*, and DC equal to *n*, join BC, draw DE parallel to CB, and E will be the point required.

For AD : DC ∷ AE : EB ——— ——— 185
And AD = *m*, DC = *n* —— —— —— con.
Th. *m* : *n* ∷ AE : EB ——— ——— —— 56

Pro-

Problem 14.

To find a third proportional to two given lines A B, B C;

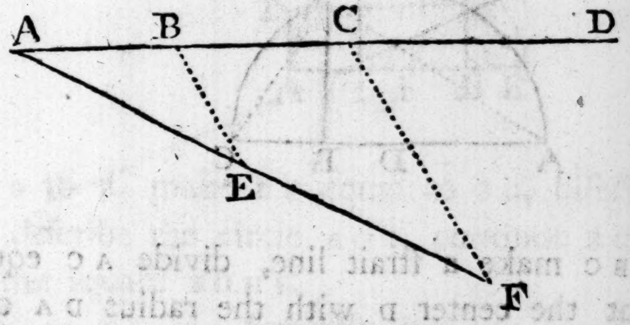


Let A B and B C make a ſtrait line, draw A E at pleaſure, and in it take A D equal to B C by problem 3, join B D, draw C E parallel to B D, and D E will be the line required.

For AB : BC :: AD : DE ——— ——— 185
And AD is equal to B C ——— ——— con.
Th. AB : BC :: BC : DE ——— ——— 56

Problem 15.

To find a fourth proportional to three given ſtrait lines, AB, BC, CD.

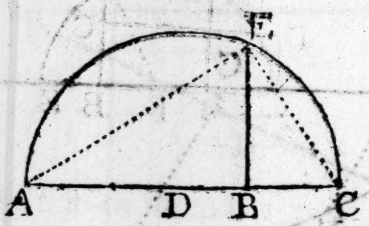


Let AB, BC make a ſtrait line, draw AF at pleaſure, and in AF take AE equal to CD, join BE, draw CF parallel to BE, and EF will be the line required.

For AB : BC :: AE : EF ——— ——— : ——— 185
And AE is equal to CD. ——— ——— ——— con.
Th. AB : BC :: CD : EF ——— ——— ——— 56

Problem 16.

To find a mean proportional between two given ſtrait lines AB, BC.



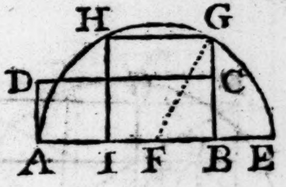
Let AB and BC make a ſtrait line, divide AC equally in D by problem 5, about the center D with the radius DA or DC deſcribe a ſemicircle AEC, and draw BE perpendicular to AC by problem 6.

For join AE, EC.

Now AEC is a right angle — — 149
And EB is perpendicular to AC — — con.
Th. AB : BE :: BE : BC — — — 196
Th. BE is a mean proportional — — 157

Problem 17.

To make a ſquare equal to a given rectangle ABCD. 148. 2.



Produce AB to E, make BE equal to BC, biſect AE in F, about the center F deſcribe the circle AGE, continue BC to G, and upon BG deſcribe the ſquare BGHI.

For join FG.

Now $AB \times BE + FB^2 = FE^2$ ——— ——— 109

And $BE = BC$, $FG = FE$ ——— ——— coh.

Th. $AB \times BC + FB^2 = FG^2$ ——— ——— 56

But ------ $BG^2 + FB^2 = FG^2$ ——— ——— 98

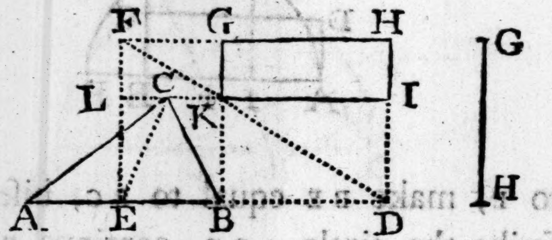
Th. $BG^2 + FB^2 = AB \times BC + FB^2$ ——— ——— 48

Th. BG^2 ------- $= AB \times BC$ ——— ——— 50

Or BGHI ------ = ABCD ——— ——— 61

Problem 18.

To make a rectangle upon a given line GH, that ſhall be equal to a given triangle ABC.



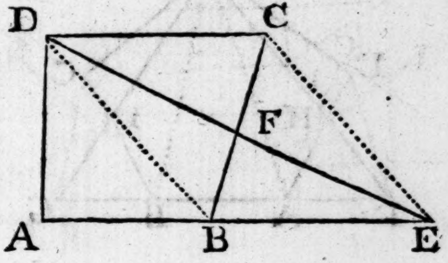
Produce AB to D, make BD equal to the given line, biſect AB in E, draw EF at right angles to AB and BG, DH parallel to EF, draw LCI parallel to AD, cutting BG in K, join DK which continue to F, and draw FH parallel to ED, then GHIK is the rectangle required.

For join CE.

Now the triangle ACE = ECB ———— ———— 94
Th. the triangle ABC = 2ECB ———— ———— 49
But the rectangle EK = 2ECB ———— ———— 96
Th. the triangle ABC = EK ———— ———— 48
Again the rectangle GI = EK ———— ———— 97
Th. the rectangle GI = ABC ———— ———— 48
And ------------- GH = BD ———— ———— 89

Problem 19.

To make a triangle equal to a given quadrangle A B C D.



Produce A B to E, join B D, draw C E parallel to D B, and join D E.

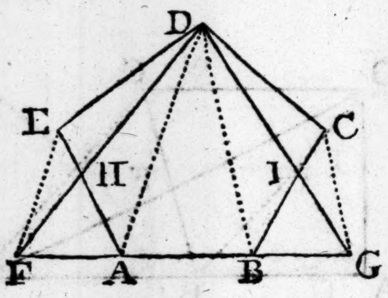
For the triangle B D E = B D C —— —— 93

Th. the triangle B E F = D C F —— —— 50

Th. the triangle A D E = A B C D —— —— 49

Problem 20.

To make a triangle equal to a given pentagon ABCDE.

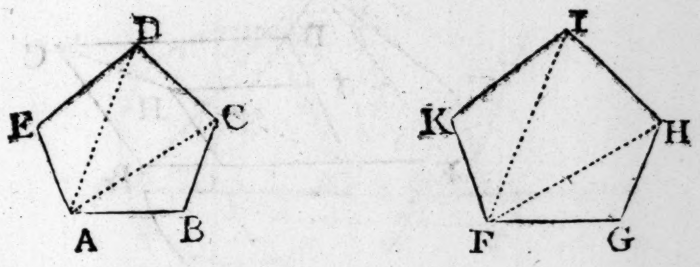


Produce the baſe AB both ways to F and G, join DA, DB, draw EF parallel to DA, CG parallel to DB, and join FD, DG.

For the triangle ADF = ADE —— —— —— 93
And the triangle BDG = BDC —— —— 93
Th. the triangle AEH = HED —— —— 50
And the triangle BGI = ICD —— —— 50
Th. the triangle FGD = ABCDE. —— —— 49

Problem 21.

Upon a given line FG, to make a figure ſimilar to given figure ABCDE.

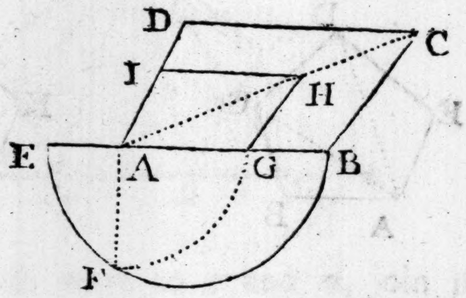


Join AC, AD, make the angle FGH equal to ABC, GFH equal to BAC, FHI equal to ACD, HFI equal to CAD, &c. by problem 9, and the thing is done.

For the angle FHG = ACB ——— ——— 85
And the angle FIH = ADC ——— ——— 85
Th. the angle GHI = BCD ——— ——— 49
And the angle HIK = CDE ——— ——— 49
Again, AC : FH :: BC : GH ——— ——— 189
And AC : FH :: CD : HI ——— ——— 189
Th. BC : GH :: CD : HI ——— ——— 169
Th. FGHIK is ſimilar to ABCDE. ——— ——— 161

Problem 22.

To make a figure ſimilar to a given figure ABCD, and in the given ratio to it of *m* to *n*.



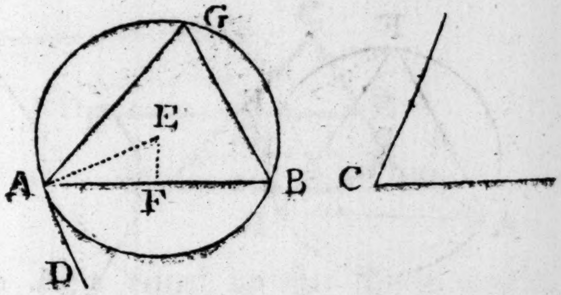
Produce BA to E, make $m : n :: BA : AE$ by problem 15, upon the diameter EB deſcribe the ſemicircle EFB, draw AF perpendicular to EB, make AG equal to AF, and the figure AGHI ſimilar to ABCD, by problem 21.

For $BA : AE :: AB^2 : AE \times AB$ —— —— 180
And AF^2, or $AG^2 = AE \times AB$ —— —— 211
Th. $BA : AE :: AB^2 : AG^2$ —— —— —— 56
Th. $m : n :: AB^2 : AG^2$ —— —— —— 169
But $ABCD : AGHI :: AB^2 : AG^2$ —— —— 208
Th. $m : n :: ABCD : AGHI$ —— —— —— 169

Problem

Problem 23.

Upon a given line A B, to deſcribe the ſegment of a circle, that ſhall contain an angle equal to a given angle C.



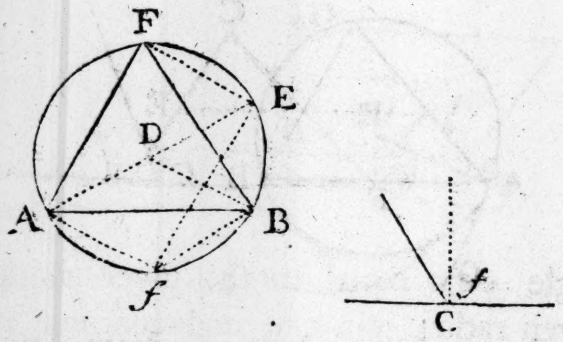
Make the angle B A D equal to C, draw A E at right angles to A D, biſect A B in F, draw F E perpendicular to A B, and about the center E, with the radius E A or E B deſcribe the ſegment A G B.

For join A G, G B.

Now A D is a tangent to the circle — 135
Th. the angle B A D = A G B — — 151
But the angle B A D = C — — con.
Th. the angle A G B = C — — — 48

23. Otherwiſe.

Upon a given line A B, to deſcribe the ſegment of a circle, that ſhall contain an angle equal to a given angle C.



Make the angles A B D, B A D equal to the difference between the given angle C, and a right angle; and about the center D, with the radius D A or D B, deſcribe the ſegment A F B.

For produce A D to E, and join A F, F B, F E.

Now A E is a diameter of the circle ——— con.

Th. the angle A F B differs from a right angle by B A E 150

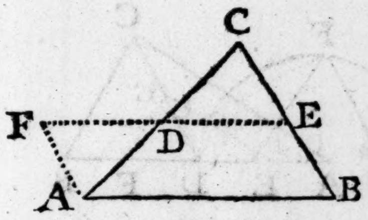
Alſo, the angle C differs from a right angle by B A E con.

Th. the angle A F B = C ——— ——— 50

Problem

Problem 24.

The baſe of a triangle, the ratio of the ſides, and the vertical angle are given to make the triangle.

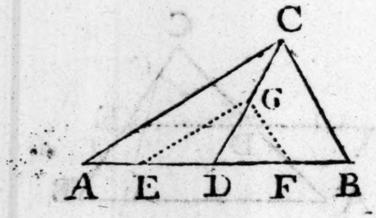


Make the angle ACB equal to the given angle, and CD to CE equal to the given ratio; join ED, and continue it to F; make EF equal to the given baſe, draw FA parallel to CB, and AB parallel to DE.

For AB is equal to EF ——— ——— 89
And DC : CE :: AC : CB —— —— —— 189
Th. ABC is the triangle required.

Problem 25.

The angles at the baſe A B, and the line C D joining the vertex and middle of the baſe are given to make the triangle.



Draw A B, and in it aſſume E D equal to D F, make E F G, F E G equal to the given angles A and B, join D G, continue it to C the given length, and draw C A, C B parallel to G E, G F.

For the triangle A C D is equiangular to E G D, and D C B to D G F —— —— —— 81

Th. C D : G D :: D A : D E —— —— 189

And C D : G D :: D B : D F —— —— 189

Th. D A : D E :: D B : D F —— —— 169

But D E is equal to D F —— —— con.

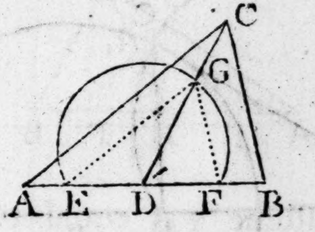
Th. D A is equal to D B —— —— 163

Th. A B C is the triangle required.

Problem

Problem 26.

The vertical angle A C B, the line C D joining the vertex and middle of the baſe, and the angle C D B made by the ſaid line and baſe are given to make the triangle.



Aſſume D E equal to D F, upon E F deſcribe the ſegment E G F, containing an angle equal to A C B by problem 23, cutting C D in G; join E G, G F, and draw C A, C B parallel to G E, G F.

For the triangle A C D is equiangular to E G D, and D C B to D G F —— —— —— —— 81.

Th. the angle A C B = E G F —— —— 49

Ag. C D : G D :: D A : D E —— —— 189

And C D : G D :: D B : D F —— —— 189

Th. D A : D E :: D B : D F —— —— 169

But D E is equal to D F —— —— con.

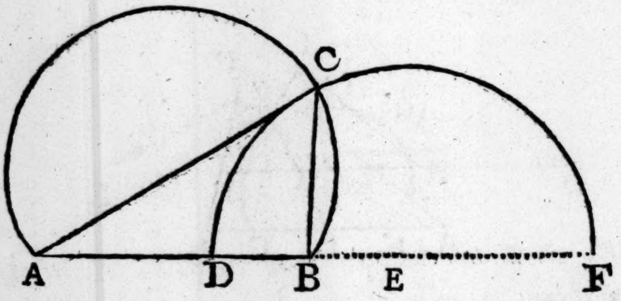
Th. A D is equal to D B —— —— 163.

Th. A B C is the triangle required.

Problem

Problem 27.

The baſe of a triangle AB, the vertical angle C, and the ratio of the ſides AC, BC as *m* to *n* are given to make the triangle.



Divide the baſe at D in the given ratio of *m* to *n*, by problem 13; produce AB to E, make AD-DB : DB :: AD : DE by problem 15, about the center E, with the radius ED deſcribe the circle DCF; upon AB deſcribe a ſegment of a circle ACB, that will contain the given angle by problem 23, and join AC, CB.

For *m* : *n* :: AD : DB ——— ——— con.

And AC : CB :: AD : DB ——— ——— 217

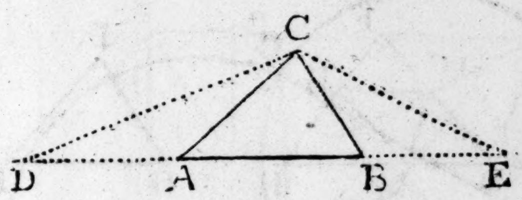
Th. *m* : *n* :: AC : CB ——— ——— ——— 169

Th. ABC is the triangle required.

Problem

Problem 28.

The angles at the baſe of a triangle, and the ſum of its three ſides are given to make the triangle.

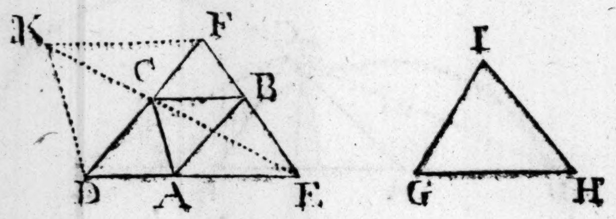


Let D E be the given ſum of the ſides: make the angles D E C, E D C equal to half the given angles at the baſe reſpectively, by problems 4 and 9; make the angle D C A equal to C D E, and the angle E C B equal D E C.

For the angle A B C = 2 B E C —— —— 84
And the angle B A C = 2 A D C —— —— 84
Th. the angles A B C, C A B are equal to the given angles 51
Again, A C = A D, and B C = B E —— —— 64
Th. A B + B C + C A = D E —— —— 49
Th. A B C is the triangle required.

Problem 29.

To inſcribe a triangle ABC in a given triangle DEF, and make it equiangular to another given triangle GHI.

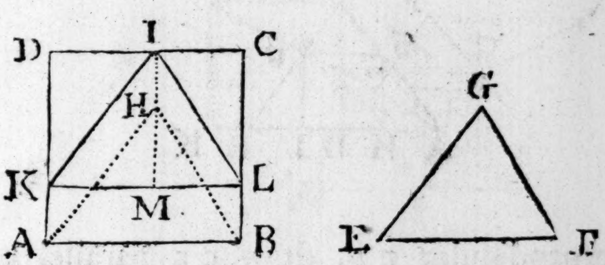


Make the angle FDK equal to G, and the angle DFK equal to H; join KE cutting DF in C, draw CA parallel to KD, CB parallel to KF, and join AB.

For the angle ACE = DKE	81
And the angle ECB = EKF	81
Th. the angle ACB = DKF	49
Ag. EK : EC :: DK : AC	189
And EK : EC :: KF : CB	189
Th. DK : AC :: KF : CB	169
Th. DK : KF :: AC : CB	171
Th. ABC is equiangular to DFK	195
But DFK is equiangular to GHI	con.
Th. ABC is equiangular to GHI	48

Problem 30.

To inſcribe a triangle in a ſquare A B C D, and make it equiangular to a given triangle E F G.



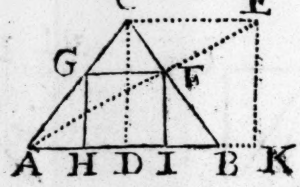
Make the angles B A H, A B H equal to the two leaſt angles of the given triangle E and F, each to each, draw H I parallel to A D, I K parallel to H A, I L parallel to H B, and join K L.

For produce I H to M.

Now K I is equal to A H ——— ——— 89
And I L is equal to H B ——— ——— 89
Alſo the angle K I M = A H M ——— ——— 81
And the angle M I L = M H B ——— ——— 81
Th. the angle K I L = A H B ——— ——— 49
Th. K L I is equiangular to A B H ——— ——— 62
But A B H is equiangular to E F G ——— con.
Th. K L I is equiangular to E F G ——— ——— 48

Problem 31.

To inſcribe a ſquare in a given triangle A B C.



Let fall the perpendicular C D, draw C E parallel to A B, make C E equal to C D, join A E cutting B C in F, draw F G parallel to B A, and G H, F I parallel to C D.

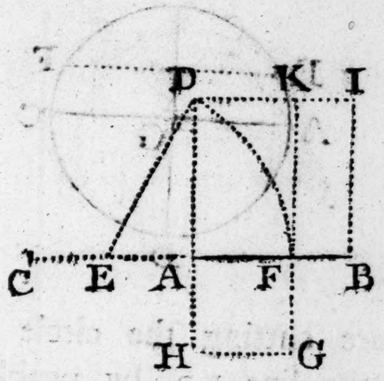
For produce A B to K, and draw E K parallel to C D.

Now AE : AF :: EC : FG	189
And AE : AF :: EK : FI	189
Th. EC : FG :: EK : FI	169
But E C is equal to C D	con.
And E K is equal to C D	89
Th. CD : FG :: CD : FI	56
Th. FG is equal to FI	163
But GH = FI, and HI = GF	89
Th. FG, GH, HI, IF are equal	48
And its angles are all right angles	90
Th. FGHI is a ſquare	24

Problem

Problem 32.

To divide a given right line AB, ſo that the rectangle contained by the whole line and one of the parts, ſhall be equal to the ſquare of the other part. 11 2. 2.



Produce BA to C, draw AD at right angles to AB, make AC and AD equal to AB, biſect AC in E, join ED, make EF equal to ED, and F is the point required.

For upon AB, AF deſcribe the ſquares of ABID, AFGH by problem 11, and produce GF to K.

Now $AD^2 + AE^2 = ED^2$ —— —— 98

And $CF \times AF + AE^2 = EF^2$ —— 112

But $ED^2 = EF^2$ —— —— —— 103

Th. $AD^2 + AE^2 = CF \times AF + AE^2$ —— 48

Th. $AD^2 = CF \times AF$ —— —— 50

But CF and DH are equal —— —— 49

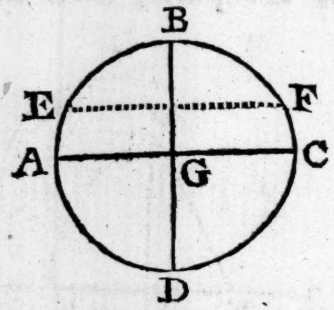
Th. $AB^2 = DH \times HG$ —— —— 56

Th. $KB = AG$, or $BI \times BF = AF^2$ —— —— 50

Th. $AB \times BF = AF^2$ —— —— 56

Problem 33.

1. 2 3 To find the centre of a given circle A B C D.



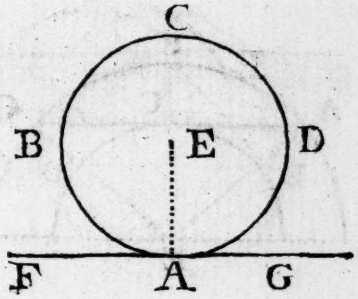
Draw E F at pleaſure cutting the circle in E and F, biſect E F at right angles with the line B D, by problem 5, and again biſect B D at right angles by the line A C in G, and the point G will be the center.

For the center is in the line B D —— —— 126
It is alſo in the line A C —— —— 126
Th. G muſt be the center of the circle.

Problem

Problem 34.

From a given point A, to draw a tangent to a given circle 1783. BCD.

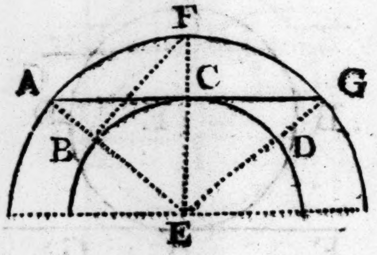


Caſe 1. when the given point is in the circumference.

Find E the center of the circle, by problem 33, join E A,
and draw A F or A G, at right angles to E A —— p. 6.
Then A F or A G, is a tangent to the circle —— 135

Problem 34.

Caſe 2. when the given point A, is without the circumference,



Find E the center of the circle, join A E, about the center E, with the diſtance E A deſcribe the circle A F G, draw B F at right angles to A E, join E F cutting the circumference in C, and draw A C.

For A E is equal to E F —— —— —— 30

E C is equal to E B —— —— —— 30

And the angle A E C is common —— —— 62

Th. the angle A C E = E B F —— —— 62

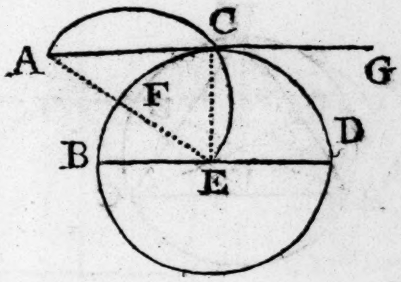
Th. A C E is a right angle —— —— 54

Th. A C is a tangent to the circle —— —— 135

Problem

34 Otherwiſe.

From the point A, to draw a tangent to the circle BCD.



Find E the center of the circle, join AE, biſect AE in F, about the center F, with the diſtance FA, deſcribe the ſemicircle AEC, and draw AC.

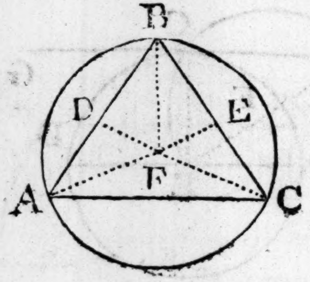
For join CE.

Now AEC is a ſemicircle —— —— con.
Th. ACE is a right angle —— —— 149
Th. AC is a tangent to the circle —— 135

Problem.

Problem 35.

To circumſcribe a circle about a given triangle ABC.



Biſect any two ſides AB, BC in D and E, draw DF, EF at right angles to AB, BC, and about the center F, with the radius FA deſcribe the circle.

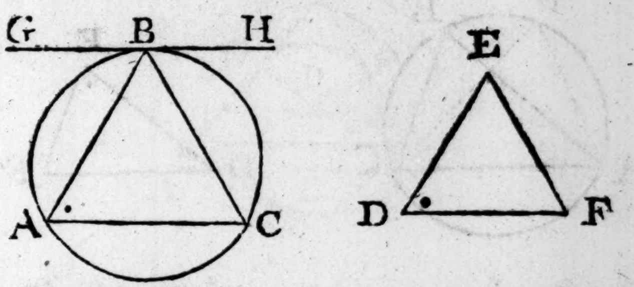
For join AF, FB, FC.

Now AD is equal to DB —— —— con.
The ſide DF is common —— —— ——
And the angle ADF = FDB —— —— con.
Th. AF is equal to FB —— —— 62
By the ſame reaſon FB = FC
Th. FA, FB, FC, are equal —— —— 48
Th. the circle paſſes through the points A, B, C —— 30

Problem

Problem 36.

In a given circle ABC, to inſcribe a triangle that ſhall be equiangular to a given triangle DEF.

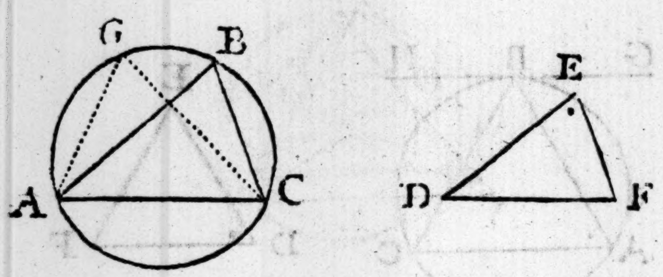


Draw GH a tangent to the circle at the point B, by problem 34, make the angle HBC equal to EDF, the angle GBA equal to DFE, and join AC.

For the angle HBC = CAB	151
And the angle GBA = ACB	151
Th. the angle CAB = EDF	48
And the angle ACB = DFE	48
Th. the angle ABC = DEF	85
Th. ABC is equiangular to DEF	27
And is inſcribed in the circle.	40

36. Otherwiſe.

In a given circle A B C, to inſcribe a triangle that ſhall be equiangular to a given triangle D E F.



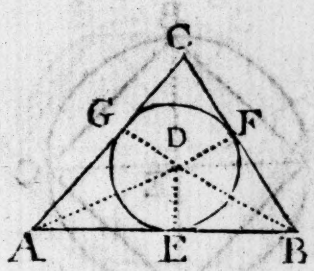
Aſſume the point G in the circumference, and join A G, make the angle A G C equal to E, draw A C, make the angle C A B equal to D, and join B C.

For the angle A G C is equal to A B C ——— 140
And the angle A G C is equal to E ——— con.
Th. the angle A B C is equal to E ——— ——— 48
Alſo the angle C A B is equal to D ——— con.
Th. the angle A C B is equal to F ——— ——— 85
Th. the triangle A B C is equiangular to D E F ——— 27

Pro-

Problem 37.

To inſcribe a circle in a given triangle A B C.

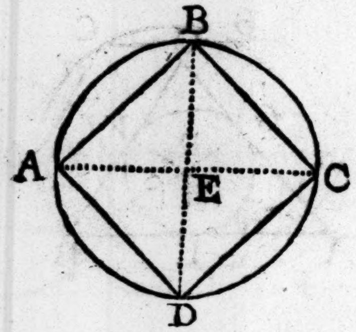


Biſect any two of the angles A B C, C A B, by the lines B D, A D, draw D E perpendicular to A B, and about the center D, with the radius D E deſcribe the circle.

For upon B C, C A let fall the perpendiculars D F, D G. p. 7
Now the angle E B D = D B F ——— ——— con.
the angle D E B = B F D ——— ——— 54
And the ſide B D is common ——— ———
Th. D E is equal to D F ——— ——— ——— 76
By the ſame reaſon D E = D G ——— ——— ———
Th. D E, D F, D G are equal ——— ——— 48
Th. the circle E F G is inſcribed in the triangle A B C. 43

Problem 38.

To inſcribe a ſquare in a given circle ABCD.



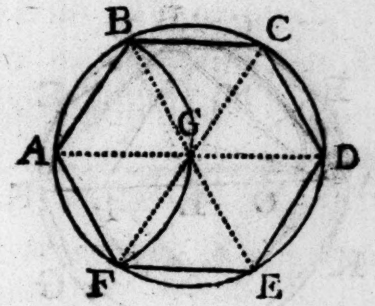
Draw the diameters AC, BD at right angles, and join AB, BC, CD, DA.

For EA, EB, EC, ED are equal ——— ——— 30
And the angles AEB, BEC, CED, DEA are equal 54
Th. AB, BC, CD, DA are equal ——— ——— 62
Ag. ABC, BCD, CDA, DAB are right angles ——— 149
Th. the oppoſite ſides are parallel ——— ——— 79
Th. ABCD is a ſquare ——— ——— ——— 24

Pro

Problem 39.

To inſcribe a hexagon in a given circle ABCD, &c.



Draw the diameter AD, find G the center of the circle, about the center A, with the radius AG, deſcribe a circle BGF, join BG, FG, produce them to E and C, and draw AB, BC, CD, DE, EF, FA.

For AB, AG, GB are equal ——— ——— con.

Th. the triangle ABG is equiangular —— —— 63

Th. each angle is $\frac{1}{3}$ of two right angles ——— 84

Th. .... AGF is $\frac{1}{3}$ of two right angles by the ſame reaſon

But the angle FGB + BGC = two right angles — 66

Th. .... BGC is $\frac{1}{3}$ of two right angles ——— 50

Th. BC is equal to AB —— ——— 62

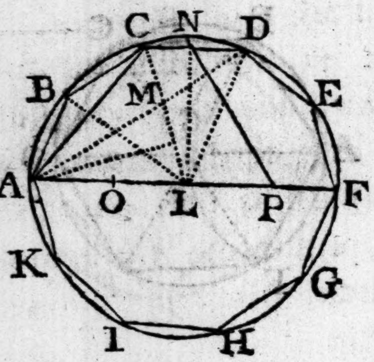
By the ſame reaſon CD, DE, &c. are equal to AB —

Th. AB, BC, CD, DE, EF, FA are equal ═══ 48

Problem

Problem 40.

To inſcribe a regular pentagon, or decagon, in a given circle.



Inveſtigation.

Let ABCD, &c. be the inſcribed decagon, join AC, AD, find L the centre of the circle, and draw BL, LC, LD.

Now the arches AB, BC, CD, &c. are equal —— 148

Th. the angles ALB, BLC, CLD are equal —— 147

And the angles ADC, DAF are equal —— 147

But the angle ADC is equal to ALB or BLC — 139

Th. the angles ADC, DAF, CLD are equal —— 48

And CD, AF are parallels —— —— 77

Th. the angles ALM, MCD are equal —— 80

Th. the triangles ALM, LCD, DCM are equiangular 85

Th. LC : CD :: DC : CM —— —— 189

And AM, ML are equal to LD, DC each to each — 76

Th. LC : LM :: LM : CM —— —— 56

Th. $LC \times CM = LM^2$ —— —— 182

Ag. AM is equal to AL ——— ——— 48
Th. $AC^2 - AL^2 = LC \times CM$ ——— ——— 119
Th. $AC^2 = AL^2 + LC \times CM$ ——— ——— 49
Th. $AC^2 = AL^2 + LM^2$ ——— ——— 56

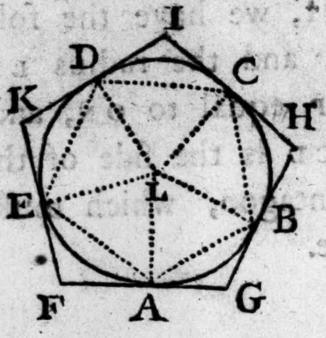
Whence by problem 31, we have the following conſtruction.

Draw the diameter AF, and the radius LN at right angles to it, biſect AL in O, make OP equal to ON, and join NP.

Then $LP = (ML =)\ CD$ is the ſide of the decagon, and $NP = AC$, is the ſide of the pentagon, which muſt be applied round the circumference of the circle.

Problem 41.

About a given circle to deſcribe a regular figure, having the ſame number of ſides with a regular figure inſcribed in that circle.



Let A, B, C, D, E be the angular points of the inſcribed figure, and at the points A, B, C, D, E, by problem 34, draw tangents to the circle, meeting one another at F, G, H, I, K.

For let L be the center of the circle.

Now the angles LEF, FAL, LAG, GBL are equal 54

And the angles LEA, LAE, LAB, LBA are equal 65

Th. the angles FEA, FAE, GAB, GBA are equal — 50

Th. the angles F, G are equal, and the ſides EF, FA, AG, GB are equal — — — — 76

By the ſame reaſon the angles G, H are equal, and AG, GB, BH, HC are equal — — —

Th. FG, GH, HI, &c. are equal — — 49

Th. the figure FGHIK is regular — — 26

And is deſcribed about the circle — — 42

FINIS.

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